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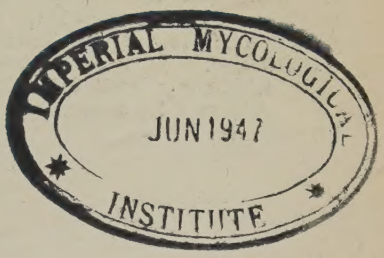

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MATHEMATICS

ON SELF-ADJOINT EXTENSIONS OF BOUNDED AND SEMI-BOUNDED HERMITIAN TRANSFORMATIONS

By M. KREIN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 3. V. 1945)

An Hermitian transformation T in the Hilbert space $\mathfrak{H}$ is said to be semi-bounded (from below), if there exists such a real constant γ that $(Tf, f) \geqslant \gamma(f, f)$ for all $f \in \mathfrak{D}(T)$ ($\mathfrak{D}(T)$ being the domain of definition of the transformation T). The exact upper bound of all possible values of γ for the given transformation T shall be denoted by $m(T)$. In his well-known paper von Neumann [1] stated the conjecture that if the domain of definition $\mathfrak{D}(T)$ of a semi-bounded transformation is dense in $\mathfrak{H}$ ($\overline{\mathfrak{D}(T)} = \mathfrak{H}$), then there exists at least one semi-bounded self-adjoint extension $\tilde{T}$ of the transformation T, for which $m(\tilde{T}) = m(T)$. This conjecture was later proved by Friedrichs [2], who left, however, the question on the uniqueness of such an extension open. We succeeded in showing that such an extension will be unique only if certain conditions are satisfied, which is not always the case. We obtain this result incidentally in solving a more general problem, namely the problem of determination of all self-adjoint extensions $\tilde{T}$ of the transformation T for which $m(\tilde{T}) \geqslant \gamma$, where γ is an arbitrarily chosen number not exceeding $m(T)$. It turns out that among such extensions there are always two «extreme» extensions T_μ and T_M, possessing a series of interesting extremal properties. The coincidence of the transformations T_μ and T_M (which is possible for a closed $T \neq T^*$, T^* being the adjoint transformation, only when $\gamma = m(T)$) means that T possesses only one self-adjoint extension $\tilde{T}$, for which $m(\tilde{T}) = m(T)$.

Since almost all Hermitian transformations occurring in problems of mathematical physics are semi-bounded, our results are capable of applications in mathematical physics.

The starting point of our method of investigation is the circumstance that the problem of extension of a semi-bounded transformation T ($\overline{\mathfrak{D}(T)} = \mathfrak{H}$) may be reduced to the problem of extension of a bounded Hermitian transformation with a domain of definition not dense in $\mathfrak{H}$.

1. In the sequel A denotes a Hermitian transformation [$(Ag, f) = (g, Af)$ for $g, f \in \mathfrak{D}(A)$], the domain of definition $\mathfrak{D}(A)$ of which is a linear closed set ($\overline{\mathfrak{D}(A)} = \mathfrak{D}(A)$) and whose norm is $\leqslant 1$ ($|A| \leqslant 1$, *i. e.* $|Af| \leqslant |f|$ for $f \in \mathfrak{D}(A)$). By $\mathfrak{B}(A)$ we denote the aggregate of all self-adjoint extensions A with a norm $\leqslant 1$ of the transformation A. The following theorem holds:

Theorem 1. *The set* $\mathfrak{B}(A)$ *is never void; moreover, it contains two transformations* A_μ *and* A_M ($A_\mu \leqslant A_M$) *such that a bounded self-adjoint transformation* B *belongs to* $\mathfrak{B}(A)$ *if and only if* $A_\mu \leqslant B \leqslant A_M$.

It is to be observed that the symbolic «inequality» $B \leqslant C$ (where B and C are two bounded self-adjoint transformations) stands for $(Bf, f) \leqslant (Cf, f)$, if $f \in \mathfrak{H}$.

2. If $Af + f \neq 0$ for $f \in \mathfrak{D}(A)$, $f \neq 0$, then the equality

$$S(f + Af) = f - Af \qquad (f \in \mathfrak{D}(A))$$

defines a certain closed Hermitian transformation S, the domain of definition of which $\mathfrak{D}(S) = (I + A)\mathfrak{D}(A)$. This transformation will be positive, *i. e.* $(Sf, f) \geqslant 0$ for $f \in \mathfrak{D}(S)$. We shall write: $S = (I - A)(I + A)^{-1}$.

Conversely, if S is a closed positive Hermitian transformation, then the equality

$$A(f + Sf) = f - Sf$$

defines the Hermitian transformation A, for which $\mathfrak{D}(A) = (I + A)\mathfrak{D}(S)$, $\overline{\mathfrak{D}(A)} = \mathfrak{D}(A)$, $|A| \leqslant 1$ and $Af + f \neq 0$ for $f \in \mathfrak{D}(A)$. We shall write:

$$A = (I - S)(I + S)^{-1}.$$

It is easily seen that $S = (I - A)(I + A)^{-1}$ if $A = (I - S)(I + S)^{-1}$.

If $\overline{\mathfrak{D}(S)} = \mathfrak{H}$, $A = (I - S)(I + S)^{-1}$, then for every $\tilde{A} \in \mathfrak{B}(A)$ will hold: $\tilde{A}f + f \neq 0$ for $f \neq 0$ and, consequently, the transformation

$$\tilde{S} = (I - \tilde{A})(I + \tilde{A})^{-1}$$

will be a positive self-adjoint extension of the transformation S. Putting

$$S_\mu = (I - A_\mu)(I + A_\mu)^{-1}, \quad S_M = (I - A_M)(I + A_M)^{-1}$$

we arrive at the

Theorem 2. *If the domain of definition* $\mathfrak{D}(S)$ *of a positive Hermitian transformation* S *is dense in* $\mathfrak{H}$, *then the set* $\mathfrak{P}(S)$ *of positive self-adjoint extensions* S *of the transformation* S *is non-void. Moreover,* $\mathfrak{P}(S)$ *contains two transformations* S_μ *and* S_M *such that a positive self-adjoint transformation* S' *will belong to* $\mathfrak{P}(S)$ *if and only if*

$$(S_\mu + aI)^{-1} \leqslant (S' + aI)^{-1} \leqslant (S_M + aI)^{-1}$$

where a *is an arbitrary positive number.*

If T is a transformation semi-bounded from below and $\overline{\mathfrak{D}(T)} = \mathfrak{H}$, then, putting $S = T - m(T)I$, $\tilde{T} = \tilde{S} + m(T)I$, where $\tilde{S} \in \mathfrak{P}(S)$, we obtain that $m(\tilde{T}) = m(T)$, which proves Friedrichs' theorem mentioned above.

3. In the sequel we shall assume that S is a positive closed Hermitian transformation such that $\overline{\mathfrak{D}(S)} \neq \mathfrak{H}$, $S \neq S^*$; by $\mathfrak{N}_a$ ($a > 0$) we shall denote the set of all solutions of the equation

$$S^* \varphi + a\varphi = 0$$

The value of a (> 0) shall be arbitrarily fixed.

Theorem 3. *Let*

$$\tilde{S}f = \int_0^\infty \lambda \, dE(\lambda) f \qquad (f \in \mathfrak{D}(\tilde{S}))$$

be the spectral representation of the transformation $\tilde{S} \in \mathfrak{P}(S)$. *In order that* $\tilde{S} = S_\mu$ ($\tilde{S} = S_M$), *it is necessary and sufficient that for every* $\varphi \in \mathfrak{N}_a$ *the integral* $J_1(\varphi; \tilde{S}) = \int_0^\infty \lambda \, d|E(\lambda) f|^2$ ($J_{-1}(\varphi; \tilde{S}) = \int_0^\infty \frac{1}{\lambda} \, d|E(\lambda) f|^2$ *should be infinite.*

In particular, the transformation $\tilde{S}$ will be the unique positive self-adjoint extension of the transformation S if and only if $J_1(\varphi; \tilde{S}) = J_{-1}(\varphi; \tilde{S}) = \infty$ for every $\varphi \in \mathfrak{N}_a$.

It may be easily shown that the condition $m(S) = 0$ is necessary for the set $\mathfrak{P}(S)$ to consist of only one transformation; this condition, however, is not sufficient. In fact, if H is a positive unbounded transformation, the continuous spectrum of which contains the point 0, then there is always a vector $\varphi \in \mathfrak{H}$ such that $J_1(\varphi; H) = \infty$, $J_{-1}(\varphi; H) < \infty$.

Denote by S the Hermitian transformation which is obtained from H if its domain of definition $\mathfrak{D}(H)$ is narrowed down to a set $\mathfrak{D}(S)$ consisting only of those $f \in \mathfrak{D}(H)$, for which $(Hf + af, \varphi) = 0$. It may be easily shown that $\overline{\mathfrak{D}(S)} = \mathfrak{H}$, $m(S) = 0$, $S \neq S^*$ (the transformation S has the deficiency index (1,1)). At the same time the set $\mathfrak{P}(S)$ will contain, by Theorem 3, beside the transformation $H = S_\mu$, other transformations, for instance, $S_M (\neq S_\mu)$.

The following criterion is interesting because it does not require the knowledge of at least one $\tilde{S} \in \mathfrak{P}(S)$.

Theorem 4. *In order that $\mathfrak{P}(S)$ consist of only one transformation, it is necessary and sufficient that for every $\varphi \in \mathfrak{N}_a$ $(\varphi \neq 0)$ should hold the condition*

$$\inf_{f \in \mathfrak{D}(S)} \frac{(Sf, f)}{|(f, \varphi)|^2} = 0 \tag{1}$$

4. Let T be a transformation semi-bounded from below. We shall say that the sequence $\{f_n\} \in \mathfrak{D}(T)$ is T-convergent and has for its T-limit the element $f \in \mathfrak{H}$, if $f_n \to f$ for $n \to \infty$ and $(T(f_m - f_n), f_m - f_n) \to 0$ for $m, n \to \infty$.

Denote by $\mathfrak{D}[T]$ the set of all T-limits of all possible T-convergent sequences $\{f_n\} \subset \mathfrak{D}(T)$. Evidently $\mathfrak{D}(T) \subset \mathfrak{D}[T]$*. Friedrichs has shown that if the sequences $\{g_n\} \subset \mathfrak{D}(T)$ and $\{f_n\} \subset \mathfrak{D}(T)$ converge respectively to g and f, then there always exists the limit

$$\lim_{n \to \infty} (Tg_n, f_n) \tag{2}$$

and this limit does not depend on the choice of the sequences $\{g_n\}$ and $\{f_n\}$, if only they have the same T-limits g and, respectively, f. The limit (2) we shall denote by $T[g, f]$. Thus, the bilinear Hermitian form $T[g, f]$ is defined for all $g, f \in \mathfrak{D}[T]$.

It may be easily shown that two self-adjoint semi-bounded transformations T_1 and T_2 coincide if and only if

$$\mathfrak{D}[T_1] = \mathfrak{D}[T_2] \quad \text{and} \quad T_1[f, f] = T_2[f, f] \quad \text{for} \quad f \in \mathfrak{D}[T_1]$$

Theorem 5. *Among all semi-bounded from below extensions $\tilde{S}$ of the transformation S only the extension S_μ possesses the property that $\mathfrak{D}(\tilde{S}) \subset \mathfrak{D}[S]$. Besides $\mathfrak{D}[S_\mu] = \mathfrak{D}[S]$ and*

$$S_\mu[g, f] = S[g, f] \quad \text{for} \quad g, f \in \mathfrak{D}[S]$$

From Theorem 5 it follows, as anyone acquainted with Friedrichs' paper will see, that the method of extension of the transformation S to a self-adjoint transformation $\tilde{S}$, which was proposed by this author, always leads to the transformation S_μ.

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* Observe that in virtue of theorem 5 $\mathfrak{D}[S]$ is a true part of $\mathfrak{D}[\tilde{S}]$.

Theorem 6. *For every* $\tilde{S} \neq S_\mu$ $(\tilde{S} \in \mathfrak{P}(S))$

$$\mathfrak{D}[S] \subset \mathfrak{D}[\tilde{S}] \subset \mathfrak{D}[S_M]$$

and

$$\tilde{S}[f, f] = S_M[f, f] \quad \textit{for} \quad f \in \mathfrak{D}[\tilde{S}]^*$$

It is also evident that

$$\tilde{S}[f, f] = S[f, f] \quad \textit{for} \quad f \in \mathfrak{D}[S]$$

5. We proceed now to show, how the transformation S_μ may be constructed, if any semi-bounded from below extension $\tilde{S}$ of the transformation S is given (for want of space we omit the construction of the transformation S_M). Since for any $a > 0$ $(S + aI)_\mu = S_\mu + aI$, we may without loss of generality suppose that $m(\tilde{S}) > 0$. Denote by $\mathfrak{N}_0$ the set of all solutions of the equation $S^* \varphi = 0$. If $\tilde{S} \neq S_\mu$, then by Theorem 3 the set $\mathfrak{D}[\tilde{S}]$ has a non-void intersection with the set $\mathfrak{N}_0^{**}$; let us denote it by $\mathfrak{L}$ $(\mathfrak{L} = \mathfrak{N}_0 \cap \mathfrak{D}[\tilde{S}])$. We shall suppose that the dimensionality of $\mathfrak{L}$ is not higher than the power of a denumerable set. Since $\tilde{S}[f, f] \geqslant m(\tilde{S})(f, f)$ $(f \in \mathfrak{D}[\tilde{S}])$, we may, under this assumption, always construct a finite or infinite $\tilde{S}$-orthonormed sequence $\{\varphi_k\} \subset \mathfrak{L}$:

$$\tilde{S}[\varphi_j, \varphi_k] = \begin{cases} 1 & \text{for } j = k \\ 0 & \text{for } j \neq k \end{cases}$$

such that it is $\tilde{S}$-complete in $\mathfrak{L}$.

Then we have

$$S_\mu^{-1} f = \tilde{S}^{-1} f - \sum (f, \varphi_j) \varphi_j \quad (f \in \mathfrak{H})$$

The obtained result possesses interesting applications to problems of mathematical physics. In a more detailed exposition of our investigations we shall show that this result enables us to construct from the Green function of an Hermitian differential operator, corresponding to some self-adjoint boundary conditions, the Green function of the same operator corresponding to some different self-adjoint boundary conditions.

6. In conclusion we may point out that in the case, where the transformation S has the deficiency-index (1,1), the combination of the results of the present note with our preceding investigations [3] on Hermitian transformations with deficiency-indices (1,1) enable us to develop a generalized problem of moments [4] on a finite interval or the half-axis.

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Academy of Sciences of the Ukrainian SSR.

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3.V.1945.

REFERENCES

[1] J. v. Neumann, Math. Ann., **102**, H. 1, 103 (1929). [2] K. Friedrichs, Math. Ann., **109**, H. 4 (1934). [3] M. Krein, C. R. Acad. Sci. URSS, XLIII, No. 8 (1944); XLIV, No. 4 (1944); XLIV, No. 5 (1944). [4] M. Krein, ibid., XLIV, No. 6 (1944).

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* For, as is easily shown, $f \in \tilde{\mathfrak{D}}[\tilde{S}]$ if and only if $J_1(f, \tilde{S}) < \infty$, and in this case $J_1(f, \tilde{S}) = \tilde{S}[f, f]$.

** If $m(S) > 0$, then S'_M is characterized also by that the equation $S_M \varphi = 0$ has precisely the same solutions as the equation $S^* \varphi = 0$.

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MATHEMATICS

ON WEYL'S PROJECTIVELY EUCLIDEAN SPACE

By A. NORDEN

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It is known that Weyl's space of the number of dimensions $n > 2$, admitting of a geodesic representation on the Euclidean space*, is Riemannian of constant curvature. Weyl (1) has shown that only for $n = 2$ there exists a non-trivial class of spaces of the indicated type, but did not express the quantities determining it in a finite form.

In the present note the problem of such a determination of $(PW)_2$ is solved geometrically by using the projective interpretation of projectively Euclidean spaces proposed by Bortolotti (2).

According to Bortolotti, to every projectively Euclidean space of n dimensions may be correlated a certain dual correspondence, *i. e.* a continuous correspondence between the points and the hyperplanes of the n-dimensional projective space non-incident with them, containing rectilinear representations of geodesics.

The main result, solving the formulated problem, reads as follows: *for the construction of the dual correspondence giving after Bortolotti the interpretation of* $(PW)_2$, *it is necessary to define in the projective plane two arbitrary «absolute» curves and to correlate to the point of intersection of their tangents the straight line joining the points of contact.*

Defining the absolute curves by the equations

$$z = z(\sigma); \quad \bar{z} = \bar{z}(\bar{\sigma}) \tag{1}$$

where z and $\bar{z}$ are the abbreviated denotations of the homogeneous coordinates of their points, and σ and $\bar{\sigma}$ are parameters, we obtain in the domain of one-valued filling-up by the points of intersection of the tangents the connectivity coefficients $(PW)_2$ of the following form:

$$\Gamma^1_{11} = \frac{\partial}{\partial\sigma}\left(\frac{A}{A^2}\right); \quad \Gamma^2_{22} = \frac{\partial}{\partial\bar{\sigma}}\left(\frac{\bar{A}}{\bar{A}^2}\right); \quad \Gamma^0_{11} = \Gamma^1_{22} = \Gamma^1_{12} = 0 \tag{2}$$

Hereby the quantities A and $\bar{A}$ are expressed through determinants composed of homogeneous coordinates and their derivatives by the parameters in the following way:

$$A = -(z\,\bar{z}\,\dot{\bar{z}}); \quad \bar{A} = -(z\,\dot{z}\,\ddot{z}) \tag{3}$$

if the coordinates are normed so that

$$(z\,\dot{z}\,\ddot{z}) - (\bar{z}\,\dot{\bar{z}}\,\ddot{\bar{z}}) = 1 \tag{4}$$

* In the sequel we shall denote the spaces of this type by $(PW)_n$.

But if the coordinates are normed and parametrized in such a way that

$$z\{\sigma, w(\sigma); 1\};\ \bar{z}\{\bar{\sigma}, \bar{w}(\bar{\sigma});\ 1\} \tag{5}$$

then

$$\begin{aligned} A &= \bar{\gamma}^2 \gamma [\dot{w}(\bar{\sigma}-\sigma)+w-w] \\ \bar{A} &= \gamma^2 \bar{\gamma} [\dot{w}(\sigma-\bar{\sigma})+w-\bar{w}] \end{aligned} \tag{6}$$

where

$$\gamma = (\dot{\dot{w}})^{-1/3};\ \bar{\gamma} = (\dot{\bar{w}})^{-1/3}$$

Hence follows that the connectivity of $(PW)_2$ depends on two arbitrary functions of one argument. In order to define $(PW)_2$ in a general system of coordinates, we must characterize the tensors g_{ij} and ω_i, coonected by the fundamental relation

$$\nabla_k g_{ij} = \omega_k g_{ij} \tag{7}$$

The first of them determines, if correspondingly normed, the canonic metrics of Riemann satisfying the condition

$$\Delta\varphi + 3a \sin\varphi = 0 \tag{8}$$

where $\varphi = \frac{1}{2}$ arccos (k/a); k is the curvature of the canonic metrics; a, a constant; and Δ, the symbol of the second differential parameter of Beltrami. The tensor ω_k is obtained by a rotation through a right angle of doubled gradient of the scalar φ. The geometrical meaning of this scalar consists in that it is equal to the angle between two vectors, one of which is parallelly translated in the canonic metrics and the other in the translation of the given $(PW)_2$.

In the coordinates σ, $\bar{\sigma}$ the fundamental form of canonic metrics possesses the form

$$\Phi = \frac{2(\dot{\dot{w}}\,\dot{\bar{w}})^{1/2}\, d\sigma\, d\bar{\sigma}}{a\{\dot{w}(\bar{\sigma}-\sigma)+w-w\}^{1/2}\{\dot{\bar{w}}(\sigma-\bar{\sigma})+\bar{w}-w\}^{1/2}} \tag{9}$$

But if we take for the absolute curves two complex conjugated analytic curves, related to complex conjugated parameters, then

$$\Phi = \frac{2|\dot{w}|\, d\sigma\, d\bar{\sigma}}{a|\dot{w}(\bar{\sigma}-\sigma)+w-\bar{w}|} \tag{10}$$

For $w = \sigma^2$ the absolute is a curve of the second degree, the curvature of the form Φ is constant, the vector $\omega_k = 0$ is equal to zero, and $(PW)_2$ is a Riemannian space of constant curvature.

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REFERENCES

[1] H. Weyl, Göttinger Nachrichten, S. 99 (1921). [2] E. Bortolotti, Annali di matematica p. ed ap., **11** (1932).

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MATHÉMATIQUES

GÉNÉRALISATION DES THÉORÈMES DE R. JENTZSCH ET F. GANTMACHER

By T. A. SARYMSAKOV, de l'Académie des Sciences d'Uzbekistan

Dans la présente Note nous donnons une généralisation des théorèmes de Jentzsch (1) et Gantmacher (2) dans la théorie des équations intégrales linéaires homogènes.

Théorème 1. *Considérons l'équation intégrale*

$$\varphi(x) = \lambda \int_a^b K(x, s)\, \varphi(s)\, d\sigma(s) \tag{1}$$

où $\sigma(s)$ est une fonction croissante, le noyau $K(x, s)$ est continu et non négatif pour $a \leqslant x, s \leqslant b$. Pour que l'équation intégrale (1) possède une solution positive $\varphi_0(x)$ correspondant à la valeur caractéristique λ_0, qui est la seule racine simple positive et de module minimum pour l'équation $D(\lambda) = 0$, il est nécessaire et suffisant qu'il existe un entier positif $n_0 > 0$ tel que la m-ième itération du noyau

$$K^{(m)}(x, x) > 0 \quad (a \leqslant x \leqslant b)$$

soit positive pour toutes les valeurs entières de m vérifiant l'inégalité $m \geqslant n_0$.*

Le théorème suivant présente aussi intérêt.

Théorème 2. *Si l'équation intégrale $\varphi(x) = \lambda \int_a^b K(x, s)\, \varphi(s)\, d\sigma(s)$, où $\sigma(s)$ et $K(x, s)$ vérifient les conditions du théorème 1, possède une valeur caractéristique λ_0 dont le module est inférieur à celui de toutes les autres racines de l'équation $D(\lambda) = 0$, alors λ_0 vérifie l'inégalité*

$$\frac{1}{\sup \int_a^b K(x, s)\, d\sigma(s)} \leqslant \lambda_0 \leqslant \frac{1}{\inf \int_a^b K(x, s)\, d\sigma(s)} \tag{2}$$

et d'ailleurs $\inf \int_a^b K(x, s)\, d\sigma(s) > 0$.

L'inégalité (2) est analogue à une inégalité établie par Romanovsky (3) pour une racine positive dont le module ne surpasse pas celui des racines d'une certaine matrice non-négative, non-décomposable d'ordre fini. Pour les matrices positives l'inégalité correspondante a été établie par Frobenius (4).

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LITTÉRATURE CITÉE

1 R. Jentzsch, J. f. reine und angew. Mathematik, 141, H. 4, 235 (1912). 2 F. Gantmacher, C. R. Acad. Sci. URSS, I (X), No. 1, 3 (1936). 3 V. Romanovsky, Bull. Soc. Math. de France, LXI, f. III—IV, 124 (1933). 5 G. Frobenius, Sitzungsber. d. Akad. d. Wissensch., Berlin, 471 (1908).

* La suffisance de ce théorème a été démontrée par Jentzsch(1) dans la condition $K(x, s) > 0$ pour $a \leqslant x, s \leqslant b$ et par Gantmacher (2) dans la condition $K(x, s) \geqslant 0$, $K(x, x) > 0$ pour $a < x, s < b$. Ces auteurs exigent aussi que le noyau soit continu.

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HYDRODYNAMICS

ON AUTOMODEL SOLUTIONS OF EQUATIONS OF HYDRODYNAMICS POSSESSING CENTRAL SYMMETRY

By K. P. STANIUKOVICH

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Write the system of equations in the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0 \tag{1}$$

$$\frac{\partial \ln \rho}{\partial t}+u\frac{\partial \ln \rho}{\partial r}+\frac{\partial u}{\partial r}+\frac{Nu}{r}=0 \tag{2}$$

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial r}=0 \tag{3}$$

$$p=\rho^{\gamma}S \tag{4}$$

Herein p denotes the pressure; ρ, the density; $\gamma=c_p/c_v$; S is a quantity connected with the entropy θ by the equation $\theta=\text{const}\ln S$; $N=0$ in the case of one-dimensional motion, $N=1$ in the case of plane motion, $N=2$ in the case of motion possessing spherical symmetry.

Thus, we consider such a motion, in which the entropy is preserved in the particle, but may vary from one particle to another.

We shall solve these equations under the assumption that

$$u=(t+\tau)^{\alpha_1-1}\xi(z),\quad \rho^{\gamma-1}=(t+\tau)^{\alpha_2}\eta(z),\quad S=(t+\tau)^{\alpha_3}\sigma(z)$$

where $z=r(t+\tau)^{-\alpha_1}$, and τ is an arbitrary constant. The substitution with the original equations yields:

$$\xi'[a_1z-\xi]=(a_1-1)\xi+\gamma\sigma\left[\frac{\gamma}{\gamma+1}\frac{\eta'}{\eta}+\frac{\sigma'}{\sigma}\right] \tag{1a}$$

$$\frac{\eta'}{\eta}[a_1z-\xi]=a_2+(\gamma-1)\left[\xi'+N\frac{\xi}{z}\right] \tag{2a}$$

$$\frac{\sigma'}{\sigma}[a_1z-\xi]=a_3 \tag{3a}$$

where the quantities a_1, a_2 and a_3 must be connected by the relation

$$a_3=2(a_1-1)-a_2 \tag{4a}$$

Let us introduce the variables

$$x=\xi/z \tag{5}$$

$$y=\eta\sigma/z^2 \tag{6}$$

The equations (1a), (2a), (3a) take then the form

$$(x+x'z)(a_1-x)=(a_1-1)x+y\left[\frac{\gamma}{\gamma-1}\frac{\eta'z}{\eta}+\frac{\sigma'z}{\sigma}\right] \tag{1b}$$

$$\frac{\eta'z}{\eta}(a_1-x)=a_2+(\gamma-1)[x+x'z+Nx] \tag{2b}$$

$$\frac{\sigma'z}{\sigma}(a_1-x)=a_3. \tag{3b}$$

Take the logarithmic derivative with respect to z of the relation (6):

$$\frac{\eta'z}{\eta}+\frac{\sigma'z}{\sigma}=\frac{y'z}{y}+2. \tag{6a}$$

Determining from (2b) and (3b) $\frac{\eta'}{\eta}z$ and $\frac{\sigma'}{\sigma}z$ and substituting the values obtained into (1b) and (6a), we find

$$y=\frac{(a_1-x)[x(1-x)+(a_1-x)x'z]}{2(a_1-1)+\frac{a_2}{\gamma-1}+\gamma(N+1)x+\gamma x'z} \tag{7}$$

$$\frac{dy}{y}=\frac{(\gamma-1)dx}{a_1-x}+\frac{[N(\gamma-1)+(\gamma+1)]x-2}{a_1-x}\frac{dz}{z} \tag{8}$$

Eliminating dz/z from (7) and (8), we obtain the equation

$$d\ln z=\frac{(a_1-x)dy-(\gamma-1)ydx}{y[(N(\gamma-1)+(\gamma+1)x-2]}=$$

$$=\frac{dx[(a_1-x)^2-\gamma y]}{y\left[2(a_1-1)+\frac{a_2}{\gamma-1}+\gamma(N-1)x\right]-x(1-x)(a_1-x)} \tag{9}$$

The solution may evidently be obtained for arbitrary values of a_1 and a_2, except, however, for the case $a_1=0$; it is also evident that we may look for a solution under the assumption that

$$u=e^{a_1(t+\tau)}\xi(z),\quad \rho^{\gamma-1}=e^{a_2(t+\tau)}\eta(z),\quad S=e^{a_3(t+\tau)}\sigma(z)$$

where $z=re^{-a_1(t+\tau)}$.

Substituting these relations into the original equations, we shall have:

$$\xi'(a_1z-\xi)=a_1\xi+\eta\sigma\left[\frac{\gamma}{\gamma-1}\frac{\eta'}{\eta}+\frac{\sigma'}{\sigma}\right] \tag{1c}$$

$$\frac{\eta'}{\eta}(a_1z-\xi)=a_2+(\gamma-1)\left[\xi'+N\frac{\xi}{z}\right] \tag{2c}$$

$$\frac{\sigma'}{\sigma}[a_1z-\xi]=a_3 \tag{3c}$$

and between the a_1, a_2, a_3 holds the relation

$$a_3=2a_1-a_2. \tag{4b}$$

Introducing the variables x, y and carrying out some transformations similar to the foregoing ones, we obtain the equations

$$y=\frac{(a_1-x)[(a_1-x)x'z-x]}{2a_1+\frac{a_2}{\gamma-1}+\gamma(N+1)x+\gamma x'z} \tag{7a}$$

$$\frac{dy}{y}=\frac{(\gamma-1)dx}{a_1-x}+\frac{[N(\gamma-1)+(\gamma+1)]x}{a_1-x}\cdot\frac{dz}{z} \tag{8a}$$

$$d\ln z=\frac{(a_1-x)dy-(\gamma-1)ydx}{xy[N(\gamma-1)+\gamma+1]}=\frac{dx[(a_1-x)^2-\gamma y]}{y\left[2a_1+\frac{a_2}{\gamma-1}+\gamma(N+1)x\right]+x(a_1-x)} \tag{9a}$$

The investigation of the obtained solutions shows that they contain 5 arbitrary constants: c_1, c_2 (constants of integrations), τ, a_1 and a_2. For the one-dimensional problem ($N=0$) the solutions may be sought for in the form where $z=(r+r_0)\,(t+\tau)^{-a_1}$, or $z=(r+r_0)\,e^{-a_1(t+\tau)}$, *i. e.* we shall have the complete integral. If $S=\text{const}$, then the solutions will always yield the complete integral, since they will depend on four arbitrary constants: τ, c_1, c_2 and a_1 ($a_2=f(a_1)$).

Formally it is possible to obtain from the complete integral the general integral, but this problem leads necessarily to the integration of such a system of differential equations (obtained by variation of the arbitrary constants) which is not simpler than the original one, as has been shown by Bechert in the case of constant entropy. The equations obtained by Bechert have, in fact, a very inconvenient form for such operations, since the method by which Bechert obtains them is a rather complicated one and practically of little use for the solution of any concrete problem.

From the equations (7) and (8) it is in certain cases easy to find the first integral. Indeed, put $[N(\gamma-1)+(\gamma+1)]\,x-2=A(a_1-x)$; hence $a_2=-\frac{2}{N(\gamma-1)+\gamma+1}$, $A=-[N(\gamma-1)+\gamma+1]$, and the equation (8) yields:

$$y\,(x-a_1)^{\gamma-1}z^{[N(\gamma-1)+\gamma+1]}=c_1 \tag{10}$$

The value of c_1 may evidently be determined if the values of y and x for $t=0$ are known.

The transition to the basic variables may be performed by means of the formulae

$$x=u\frac{(t+\tau)}{r},\quad y=\rho^{\gamma-1}S\frac{(t+\tau)^2}{r^2}=\rho v\left(\frac{t+\tau}{r}\right)^2,\ \text{if}\ z=r\,(t+\tau)^{a_1},$$

$$x=\frac{u}{r},\quad y=\frac{\rho^{\gamma-1}S}{r^2}=\rho v/r^2,\ \text{if}\ z=re^{-a_1(t+\tau)}.$$

We have shown that for problems of hydrodynamics possessing similarity, the integration of the system of partial differential equations may be reduced to the integration of one usual differential equation of the first order

$$\frac{dy}{dx}=F(x,\ y) \tag{9b}$$

after which $y=y(c_1;\ c_2;\ z)$, $x=x(c_1;\ c_2;\ z)$ are to be determined from the equations (9) or (9a); and this directly determines u. Equation (3a) determines S, and then ρ and p may easily be determined.

A similar method of finding the solution may be applied to any differential equation describing some transient process, provided these equations are homogeneous in the unknown functions entering them.

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MATHEMATICAL PHYSICS

PERTURBATION METHOD WITH THE APPLICATION OF DOUBLE ITERATION

By L. S. MAYANTZ

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The method set forth below for finding the particular values and particular vectors of linear operators by means of successive approximations makes it possible to evaluate errors in the latter in a number of cases.

The method may be effected in different variants, the first approximations of one variant exactly coinciding with the first approximations of the usual perturbation method, which makes it possible in a number of cases to estimate the errors in these approximations of the usual perturbation method.

The method is set forth for operators acting in a space of a finite number of dimensions, but all the propositions are so formulated that it may also be applied (with certain reservations) to cases of operators acting in a space of infinite number of dimensions.

Suppose we have to solve the equation

$$W\vec{r} = \lambda\vec{r} \tag{1}$$

where W is an operator acting in a certain space of n dimensions; $\vec{r}$ is the vector of this space, and λ is a parameter.

Suppose further that we know the entire system of particular vectors $\vec{p_i}$ and particular values ν_i $(i = 1, 2, \ldots, n)$ of a certain operator W_0 acting in the same space.

Let us rewrite (1) in the form

$$W'\vec{r} = \lambda\vec{r} - W_0\vec{r}$$

where $W' = W - W_0$ is the operator of the perturbation.

Taking $\vec{p_i}$ as the space basis and assuming that $W'\vec{p_i} = \sum_{j=1}^{n} w'_{ji}\vec{p_j}$, we obtain, since $W_0\vec{p_i} = \nu_i\vec{p_i}$, the system of equations:

$$r^i(\lambda - \nu_i) = \sum_{j=1}^{n} w'_{ij}r^j \quad (i = 1, 2, \ldots, n) \tag{2}$$

where r^i are the components of $\vec{r}$.

The solutions of (2) give the particular vectors $\vec{r}'_i = \sum_{j=1}^{n} r^i_{j'} \vec{p}_j$ and the particular values $\lambda_{i'}$ $(i' = 1, 2, \ldots, n)$ of the operator W.

Let us introduce the notation $\gamma_i - \nu_i - w'_{ii} = \Delta_i$. Then for a certain $i = l$

$$\Delta_l = \sum_{i \neq l} w'_{li} \frac{r^i_{l'}}{r^l_{l'}} \tag{3a}$$

$$(\Delta_l + w'_{ll} - w'_{ii} + \nu_l - \nu_i) \frac{r^i_{l'}}{r^l_{l'}} = w'_{il} + \sum_{j \neq l, i} w'_{ij} \frac{r^j_{l'}}{r^l_{l'}} \quad (i \neq l). \tag{3}$$

Let us introduce the vector r' with components $r'^i = \frac{r^i_{l'}}{r^l_{l'}} (i \neq l)$; $\vec{w}$ with components $w^i = w'_{il}$ $(i \neq l)$; w' with components $w'^i = w'_{li}$ $(i \neq l)$; and the matrix $B = \| b_{ij} \|$ with $b_{ij} = -w'_{ij}$ $(i \neq j)$ and $b_{ii} = \Delta_l + w'_{ll} - w'_{ii} + \nu_l - \nu_i$ $(i, j \neq l)$.

We now rewrite (3a) and (3b) in the following form:

$$\Delta = (\vec{w}', \vec{r}')^* \tag{3a'}$$

$$B\vec{r}' = \vec{w} \tag{3b'}$$

and introduce the sequences Δ_s and $\vec{r}'_s$ $(s = 1, 2, \ldots)$ defined by the expressions:

$$\Delta_s = (\vec{w}', \vec{r}'_{s-1}) \tag{3a''}$$

$$B_s \vec{r}'_s = \vec{w} \tag{3b''}$$

where $B_s = B - (\Delta - \Delta_s) E$; $\vec{r}'_0 = 0$.

Let us assume further that $|B_s|$ are different from zero for all values of s. Then B_s may be represented for all values of s by some method in the form $B_s = \| d_{s(i)} \delta_{ij} \| (E - A_s)$**.

We now rewrite (3b'') in the form:

$$\vec{r}'_s = \vec{w}''_s + A_s \vec{r}'_s \tag{3b'''}$$

where $\vec{w}''_s = \left\| \frac{1}{d_{(s)i}} \delta_{ij} \right\| \vec{w}$, and consider the sequences $\Delta_{s+1,\mu}$ and $\vec{r}_{s,\mu}$ $(\mu = 0, 1, 2, \ldots)$, defined by the expressions:

$$\Delta_{s+1,\mu} = (\vec{w}', \vec{r}'_{s\mu}); \quad \vec{r}'_{s,\mu+1} = \vec{w}''_s + A_s \vec{r}'_{s,\mu}; \quad \vec{r}'_{s,0} = \vec{r}_{s-1}.$$

If the $\lim_{\mu \to \infty} \vec{r}'_{s,\mu} = \vec{r}'_s$ for all s and the $\lim_{s \to \infty} \Delta_s = \Delta$, then $\lim_{s \to \infty} (\lim_{\mu \to \infty} \vec{r}'_{s,\mu}) = \vec{r}'$ and $\lim_{s \to \infty} (\lim_{\mu \to \infty} \Delta_{s+1,\mu}) = \Delta$, *i. e.* $\vec{r}'_{s,\mu}$ and $\Delta_{s+1,\mu}$ are approximate solutions of equations (3a') and (3b'), differing as little as desired from the exact solution of these equations.

* For convenience in notation the index l is hereafter dropped from Δ.

** The different variants of the method pointed out above are obtained by choosing different values for $d_{(s)i}$, the choice of $d_{(1)i} = \varphi_l - \varphi_i$ leading to $r_{1,1}$ and $\Delta_{1,2}$ (see below) coinciding with the second approximation, and $r_{1,2}$ and $\Delta_{2,2}$ with the third approximation of the usual perturbation method.

In order that the $\lim_{\mu\to\infty} \vec{r}'_{s,\mu} = \vec{r}'_s$, it is necessary and sufficient that at a certain $k \geqslant 1$ some sufficient condition of convergence of the corresponding Neuman series be fulfilled for the matrix $C_s = A_s^k$, for example (1):

$$s_1 = \sum_j |C_{(s)ij}| \leqslant M_s < 1 \qquad \text{(for all } i\text{)} \tag{a}$$

In order that the $\lim_{s\to\infty} \Delta_s = \Delta$, it is sufficient, as is known (2), that the derivative of the right-hand side of (3a′) with respect to Δ be less than unity in the interval from 0 to Δ^0, satisfying (3a′), if it is positive in this interval, or its absolute value should be less than unity in the interval from 0 to $2\Delta^0$, if it is negative in this interval.

To evaluate the errors in the approximations obtained, we first introduce certain notations:

$$\Delta - \Delta_s = \varepsilon_s;\ \Delta - \Delta_{s+1,\mu} = \varepsilon_{s+1,\mu};\ \Delta_{s+1} - \Delta_{s+1,\mu} = \varepsilon_\mu^{s+1};$$
$$\Delta_{s+1} - \Delta_s = \eta_s;\ \Delta_{s+1,\mu} - \Delta_s = \eta_s^\mu;\ \vec{r}' - \vec{r}'_s = \vec{\rho}_s;\ \vec{r}' - \vec{r}'_{s,\mu} = \vec{\rho}_{s,\mu};$$
$$\vec{r}'_s - \vec{r}'_{s,\mu} = \vec{\rho}_\mu^{(s)};\ \vec{r}_{s,\mu} - \vec{r}'_{s,\mu-1} = \vec{w}''_{s,\mu}.$$

Then $\varepsilon_{s+1,\mu} = \varepsilon_{s+1} + \varepsilon_\mu^{(s+1)}$ and $\vec{\rho}_{s,\mu} = \vec{\rho}_s + \vec{\rho}_\mu^{(s)}$. Hence

$$|\varepsilon_{s+1,\mu}| \leqslant |\varepsilon_{s+1}| + |\varepsilon_\mu^{(s+1)}|,\quad |\rho_{s,\mu}^i| \leqslant |\rho_s^i| + |\rho_\mu^{(s)i}|$$

where $\rho_{s,\mu}^i$; ρ_s^i and $\rho_\mu^{(s)i}$ are the components of the corresponding vectors.

It is easy to prove that if (a) is realized, then

$$|\rho_\mu^{(s)i}| \leqslant \frac{|w''^m_{s,\mu}|}{1 - M_s} \sum_j |t_{(s)ij}| = \rho'_{(s,\mu,i)} \tag{I′}$$

where $t_{(s)ij}$ are the elements of the matrix $T_s = \sum_{l=1}^{k} A_s^l$; $w''^m_{s,\mu}$ is the largest component of $\vec{w}''_{s,\mu}$ in absolute value and M_s is determined from condition (a). Whence

$$|\varepsilon_\mu^{s+1)}| \leqslant \frac{|w''^m_{s,\mu}|}{1 - M_s} \sum_i |w_i| \sum_j |t_{(s)ij}| = \varepsilon'_{(s+1,\mu)} \tag{II′}$$

The estimate of $|\varepsilon_{s+1}|$ and $|\rho_s^i|$ may be obtained from

$$\varepsilon_{s+1} = (\vec{w}', \vec{\rho}_s),\ \vec{\rho}_s = -(\varepsilon_{s+1} + \eta_s) B_s^{-1} [E + (\varepsilon_{s+1} + \eta_s) B_s^{-1}]^{-1} \vec{r}'_s,$$

making use of the fact that if the smallest value of ε_{s+1} satisfying the equation

$$\varepsilon_{s+1} = f_1(\varepsilon_{s+1});\ 0 \leqslant f_1(\varepsilon_{s+1})\ (0 \leqslant \varepsilon_{s+1} \leqslant a),$$

is equal to ε'_{s+1}, then the smallest value of ε_{s+1} satisfying equation

$$\varepsilon_{s+1} = f_2(\varepsilon_{s+1});\ f_2(\varepsilon_{s+1}) \leqslant f_1(\varepsilon_{s+1})\ (0 \leqslant \varepsilon_{s+1} \leqslant a);\ 0 \leqslant f_2(0),$$

is equal to $\varepsilon''_{s+1} \leqslant \varepsilon'_{s+1}$ [$f_1(\varepsilon_{s+1})$ and $f_2(\varepsilon_{s+1})$ are assumed to be continuous in the interval from 0 to a].

For the case where $\sqrt{|\eta_s| \cdot r'_{s,m}} + \sqrt{|r'^{-m}_s| \sum_i |w'^i| M'_{s,i}} \leqslant 1$, where r'^m_s is the largest component of r'_s in absolute value, $M'_{s,i} = \frac{1}{|d_{(s)i}|} + \frac{1}{1 - M_s} \sum_j \frac{|t_{(s)mj}|}{|d_{(s)j}|}$,

$M'_{s,m}$ is the largest value of $M'_{s,i}$, and $t_{(s)mj}$ is the largest element in absolute value of the j-th column of the matrix T_s

$$|\varepsilon_{s+1}| \leqslant \frac{1-|r_s^m|\sum_i |w'^i|\,M'_{is}-|\eta_s|\,M'_{s,m}}{2M'_{s,m}} - \frac{\sqrt{\left(1-|r_s^m|\sum_i |w'^i|\,M'_{s,i}+|\eta_s|\,M'_{s,m}\right)^2-4|\eta_s|\,M'_{s,m}}}{2M'_{s,m}} = \varepsilon''_{s+1} \qquad (II'')$$

$$|\rho_s^i| \leqslant |r_s'^m|\,M'_{s,i}\,\frac{\varepsilon''_{(s+1)}+|\eta_s|}{1-(\varepsilon''_{s+1}+|\eta_s|)\,M'_{s,m}} = \rho''_{(s,i)} \qquad (I'')$$

Since $|r_s'^m| \leqslant \max(|r'^i_{s,m}|+\rho'_{(s,\mu,i)}) = r_{(s,\mu)}$ and $|\eta_s| \leqslant |\eta_s^\mu| + \varepsilon'_{(s+1,\mu)} = \eta_{(s,\mu)}$, then in the final account we have:

$$|\rho^i_{s,\mu}| \leqslant \rho'_{(s,\mu,i)} + \rho'''_{(s,\mu,i)} \qquad (I)$$

$$|\varepsilon_{s+1,\mu}| \leqslant \varepsilon'_{(s+1,\mu)} + \varepsilon'''_{(s+1,\mu)} \qquad (II)$$

where $\varepsilon'''_{(s+1,\mu)}$ is obtained from $\varepsilon''_{(s+1)}$, and $\rho'''_{(s,\mu,i)}$ is obtained from $\rho''_{(s,i)}$ by replacing $|r_s'^m|$ by $r_{(s,\mu)}$ and $|\eta_s|$ by $\eta_{(s,\mu)}$.

We may point out in addition the following estimation of ε_s. If the derivative of $(\vec{w}', \vec{\rho}_{s-1})$ with respect to ε_s is not positive in the interval from 0 to η_s, then $|\varepsilon_s| \leqslant |\eta_s|$. This estimation also holds in the case where the absolute value of this derivative is greater than unity and, hence, the sequence Δ_s diverges. Such an estimation may be given, for example, in the case where the matrix $\|w'_{ij}\|$ is Hermitian.

Example. Let $\nu_1 = 1.2$; $\nu_2 = 10.5$; $\nu_3 = 11.5$;

$$\|w'_{ij}\| = \begin{Vmatrix} -0.2 & 0.2 & 0.2 \\ 0.2 & 0.5 & 0.5 \\ 0.2 & 0.5 & -0.5 \end{Vmatrix}$$

Suppose $l = 1$. Then $w'^2 = w'^3 = 0.2$, $w^2 = w^3 = 0.2$,

$$B = \begin{Vmatrix} \Delta - 10 & -0.5 \\ -0.5 & \Delta - 10 \end{Vmatrix}$$

Let us consider the approximation $s=1$; $\mu=1$. Taking $d_{(1)2} = d_{(1)^3} = -10$, we get $w''^2_1 = w''^3_1 = -0.02$ and, hence, $r'^2_{1,1} = r'^3_{1,1} = -0.02$; $\Delta_{2,1} = -0.008$.

Let us estimate the errors in this approximation. From $A_1 = \begin{Vmatrix} 0 & 0.05 \\ 0.05 & 0 \end{Vmatrix}$ we get (assuming $k=1$) $M_1 = 0.05$; $\rho'_{(1,1,2)} = \rho'_{(1,1,3)} = 0.0010$; $\varepsilon'_{(2,1)} = 0.0004$.

Further $r_{(1,1)} = 0.0210$; $\eta_{(1,1)} = 0.0084$; $M'_{1,2} = M'_{1,3} = 0.1105$, whence $\varepsilon'''_{(2,1)} = 0.0000$ and $\rho'''_{(1,1,2)} = \rho'''_{(1,1,3)} = 0.0000$. Thus, in the final account:

$$\lambda_{1'} = 0.992 \pm 0.0004, \quad \frac{r^2_{1'}}{r^1_{1'}} = \frac{r^3_{1'}}{r^1_{1'}} = -0.02 \pm 0.0010.$$

The exact solution correct to four places is:

$$\lambda_{1'} = 0.9924; \quad \frac{r^2_{1'}}{r^1_{1'}} = \frac{r^3_{1'}}{r^1_{1'}} = -0.0190.$$

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REFERENCES

[1] R. Courant and D. Hilbert, Methoden der Mathematischen Physik, 1, 1933.
[2] В. Брадис, Теория и практика вычислений, 213, 1937.

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PHYSICS

THE NUMBER OF STRUCTURAL VARIETIES IN THE SIMPLE FORMS OF CRYSTALS

By I. I. SHAFRANOVSKY

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In his two papers([1],[2]) Boky developed further the theory of simple forms in crystallography. Besides introducing greater precision into the generally accepted views concerning 47 crystallographic forms, the writer suggested a novel idea of there being physically diverse simple forms which differ from each other in their symmetry elements and in the way the latter are distributed with regard to the faces.

Boky's deduction of these forms was based on types of symmetry. As a result he obtained 146 physically diverse simple forms. Besides he discovered 47 enantiomorphous simple forms. Taking the latter into account, we thus have a total of 193 physically diverse simple forms.

A further step in the same direction was to introduce the idea of structural varieties in the simple forms of crystals. We consider two varieties of the same simple form to be structurally diverse, if they differ from each other in the symmetry elements of their space groups, or in the distribution of the latter with respect to the faces.

It should be noted that, according to Donnay and Harker([3]), the appearance of crystals is to some extent dependent on the symmetry elements of their space groups (such as the screw axes and planes of gliding reflection, as well as the simple and inverse symmetry axes). Thus, for instance, the faces which are perpendicular to screw axes or to planes of gliding reflection are of little importance from the morphological point of view (this, by the way, accounts for the fact that quartz has no pinacoid perpendicular to its triple screw axis).

The above considerations show that a knowledge of the structurally diverse varieties of crystallographic forms is interesting not only from the viewpoint of roentgeno-structural and electronographic analysis, but also from that of crystal morphology.

In counting up the number of structural varieties of the simple forms of crystals we have largely made use of the available descriptions and tables of space groups ([4]), as well as of Boky's tables of physically diverse simple forms([2]).

At first sight one may be led to believe that the number of structural varieties of any simple form is always equal to that of the space groups in which the given simple form occurs. However, a study of Table 1, which contains both the number of varieties of simple forms and that of the space groups in which they occur, clearly shows that this is not always the case.

It can be clearly seen in Table 1 that for 31 simple forms the number of structural varieties is equal to that of the space groups in which they occur (for instan-

ce, a cube forms 36 varieties and occurs in 36 space groups), while for the remaining 16 simple forms the number of varieties exceeds that of the space groups in which they occur (for instance, a pinacoid forms 213 varieties, but occurs only in 155 space groups).

This can be accounted for by the fact that not infrequently the same simple form may have several varieties occurring in the same space group, these varieties differing from each other in that their faces are differently situated with respect to the symmetry elements. Such varieties are characterized by different symbols.

Table 1

No.	Form	Number of varieties	Number of space groups	No.	Form	Number of varieties	Number of space groups
1	Pinacoid	213	155	27	Trigon-tritetrahedron	14	11
2	Tetragonal prism	126	68	28	Trigonal pyramid	13	10
3	Rhombic prism	117	65	29	Ditetragonal pyramid	12	12
4	Tetragonal dipyramid	94	48	30	Tetragonal scalenohedron	12	12
5	Monohedron	71	68	31	Pentagon-dodecahedron	12	12
6	Hexagonal prism	57	47	32	Tetrahedron	11	11
7	Ditetragonal prism	54	54	33	Tetragon-tritetrahedron	11	11
8	Dihedron	52	29	34	Tetragonal trapezohedron	10	10
9	Tetragonal pyramid	40	18	35	Hexoctahedron	10	10
10	Cube	36	36	36	Rhombic tetrahedron	9	9
11	Rhombo-dodecahedron	36	36	37	Pentagon-trioctahedron	8	8
12	Tetragon-trioctahedron	30	25	38	Trigonal trapezohedron	7	7
13	Rhombic dipyramid	28	28	39	Didodecahedron	7	7
14	Hexagonal dipyramid	27	22	40	Ditrigonal pyramid	6	6
15	Octahedron	25	25	41	Ditrigonal scalenohedron	6	6
16	Trigon-trioctahedron	25	25	42	Hexagonal trapezohedron	6	6
17	Tetrahexahedron	24	24	43	Hexatetrahedron	6	6
18	Rhombic pyramid	22	22	44	Pentagon-tritetrahedron	5	5
19	Trigonal prism	22	22	45	Ditrigonal dipyramid	4	4
20	Ditetragonal dipyramid	20	20	46	Dihexagonal pyramid	4	4
21	Dihexagonal prism	20	20	47	Dihexagonal dipyramid	4	4
22	Tetragonal tetrahedron	19	14				
23	Rhombohedron	18	15				
24	Hexagonal pyramid	18	16				
25	Ditrigonal prism	17	17				
26	Trigonal dipyramid	14	12				

Besides, the number of structural varieties is increased by forms with symbols of a common type, designated by means of letter indices, whose faces are perpendicular to the planes of gliding reflection.

Among the differently inclined faces of this type one should distinguish such planes as coincide with the component of gliding from those lying obliquely with respect to the latter.

Special tables containing a detailed description of the structural varieties of simple crystal forms are to be published in the near future, but here we shall confine ourselves to presenting the final results we have obtained.

The total of all the varieties of simple forms amounts to **1402**, which may be classified as follows:

Triclinic	2 varieties
Monoclinic	46 »
Rhombic	344 »
Tetragonal	455 »
Trigonal and hexagonal	295 »
Cubic	260 »

Table 2

No.	Form	Plane groups
1	Monohedron	C_1^I (10); C_s^I (4); C_s^{II} (4); C_s^{III} (2); C_2^I (11); C_{2v}^I (5); C_{2v}^{II} (5); C_{2v}^{III} (4); C_{2v}^{IV} (4); C_4^I (4); C_{4v}^I (2); C_{4v}^{II} (2); C_3^I (6); C_{3v}^I (2); C_{3v}^{II} (3); C_6^I (2); C_{6v}^I (1)
2	Pinacoid	C_1^I (38); C_s^I (21); C_s^{II} (23); C_s^{III} (6); C_2^I (41); C_{2v}^I (12); C_{2v}^{II} (12); C_{2v}^{III} (10); C_{2v}^{IV} (10); C_4^I (10); C_{4v}^I (3); C_{4v}^{II} (3); C_3^I (13); C_{3v}^I (3); C_{3v}^{II} (4); C_6^I (3); C_{6v}^I (1)
3	Dihedron	C_1^I (28); C_s^I (13); C_s^{II} (9); C_s^{III} (2)
4	Rhombic prism	C_s^I (23); C_s^{II} (13); C_s^{III} (3); C_1^I (78)
5	Rhombic tetrahedron	C_1^I (19)
6	Rhombic pyramid	C_1^I (22)
7	Rhombic dipyramid	C_1^I (28)
8	Tetragonal tetrahedron	C_1^I (10); C_s^I (4); C_s^{II} (4); C_s^{III} (1)
9	Tetragonal prism	C_1^I (26); C_s^I (22); C_s^{II} (16); C_s^{III} (4); C_2^I (32); C_{2v}^I (7); C_{2v}^{II} (4); C_{2v}^{III} (13); C_{2v}^{IV} (2)
10	Tetragonal pyramid	C_1^I (20); C_s^I (10); C_s^{II} (8); C_s^{III} (2)
11	Tetragonal dipyramid	C_1^I (62); C_s^I (16); C_s^{II} (14); C_s^{III} (2)
12	Ditetragonal prism	C_1^I (44); C_s^I (10)
13	Ditetragonal pyramid	C_1^I (12)
14	Ditetragonal dipyramid	C_1^I (20)
15	Tetragonal trapezohedron	C_1^I (10)
16	Tetragonal scalenohedron	C_1^I (12)
17	Trigonal prism	C_1^I (4); C_s^I (4); C_s^{II} (3); C_2^I (7); C_{2v}^I (2); C_{2v}^{III} (2)
18	Trigonal pyramid	C_1^I (7); C_s^I (3); C_s^{II} (3)
19	Trigonal dipyramid	C_1^I (10); C_s^I (2); C_s^{II} (2)
20	Rhombohedron	C_1^I (12); C_s^I (3); C_s^{II} (3)
21	Trigonal trapezohedron	C_1^I (7)
22	Ditrigonal prism	C_1^I (13); C_s^I (4)
23	Ditrigonal pyramid	C_1^I (6)
24	Ditrigonal dipyramid	C_1^I (4)
25	Ditrigonal scalenohedron	C_1^I (6)
26	Hexagonal prism	C_1^I (21); C_2^I (12); C_s^I (12); C_s^{II} (6); C_{2V}^I (3); C_{2V}^{III} (3)
27	Hexagonal pyramid	C_1^I (12); C_s^I (3); C_s^{II} (3)
28	Hexagonal dipyramid	C_1^I (21); C_s^I (3); C_s^{II} (3)
29	Hexagonal trapezohedron	C_1^I (6)
30	Dihexagonal prism	C_1^I (16); C_s^I (4)
31	Dihexagonal pyramid	C_1^I (4)

No.	Form	Plane groups
32	Dihexagonal dipyramid	C_1^I (4)
33	Tetrahedron (cubic)	C_3^I (8); C_{3v}^I (3)
34	Trigon-tritetrahedron	C_1^I (8); C_s^I (3); C_s^{II} (2); C_s^{III} (1)
35	Tetragon-tritetrahedron	C_1^I (8); C_s^I (3)
36	Pentagon-tritetrahedron	C_1^I (5)
37	Hexatetrahedron	C_1^I (6)
38	Hexahedron (cubic)	C_1^I (3); C_2^I (11); C_s^{II} (1); C_{2v}^I (5); C_{2v}^{II} (4); C_{2v}^{IV} (4); C_4^I (4); C_{4v}^I (4)
39	Tetrahexahedron	C_1^I (19); C_s^I (5)
40	Octahedron	C_3^I (20); C_{3v}^I (2); C_{3v}^{II} (3)
41	Trigon-trioctahedron	C_1^I (20); C_s^I (5)
42	Tetragon-trioctahedron	C_1^I (20); C_s^I (5); C_s^{II} (3); C_s^{III} (2)
43	Pentagon-trioctahedron	C_1^I (8)
44	Hexoctahedron	C_1^I (10)
45	Rhombo-dodecahedron	C_1^I (8); C_s^I (4); C_s^{II} (4); C_s^{III} (2); C_2^I (9); C_{2v}^I (2); C_{2v}^{II} (2); C_{2v}^{III} (4); C_{2v}^{IV} (1)
46	Pentagon-dodecahedron	C_1^I (9); C_s^I (3)
47	Didodecahedron	C_1^I (7)

Thus, there are only 392 varieties of forms occurring in the lowest group, 750 varieties in the average group and 260 varieties in the highest (or cubic) group.

Table 1 contains figures showing, respectively, the number of varieties of different simple forms and that of the space groups they occur in. The figures are arranged in decreasing order.

In counting the varieties particular attention was paid to the plane symmetry of the crystal faces.

The investigations of Fedorov, Polya and Niggli ([5], [6], [7]) have shown that the symmetry of faces considered as unilateral planes corresponds to 17 plane groups:

C_1^I (the analogue of space group $C_1^1 - P1$), C_s^I ($= C_s^1 - Pm$), C_s^{II} ($= C_s^2 - Pc$), C_s^{III} ($= C_s^3 - Cm$), C_2^I ($= C_2^1 - P2$), C_{2v}^I ($= C_{2v}^I - Pmm$), C_{2v}^{II} ($= C_{2v}^8 - Pba$), C_{2v}^{III} ($= C_{2v}^4 - Pma$), C_{2v}^{IV} ($= C_{2v}^{11} - Cmm$), C_4^I ($= C_4^1 - P4$), C_{4v}^I ($= C_{4v}^1 - P4mm$), C_{4v}^{II} ($= C_{4v}^2 - P4bm$), C_3^I ($= C_3^1 - C_3$), C_{3v}^I ($= C_{3v}^1 - C3m$), C_{3v}^{II} ($= C_{3v}^2 - H3m$), C_6^I ($= C_6^1 - C6$), C_{6v}^I ($= C_{6v}^1 - C6mm$).

(The symbols used by Niggli have been somewhat altered in accordance with those used in «International Tables» ([4])).

Table 2 contains a summary of plane symmetry groups for the faces of different simple forms. The figures in brackets show the number of the structural varieties of a given form in which one or the other symmetry group may occur.

In conclusion, Table 3 shows the number of varieties in which one or the other plane group may occur. The table contains figures showing in parallel columns for each group the number of times univalent points are found to recur.

For the C_{2v}^{IV} group we have used a rhomb as the elementary parallelogram; for trigonal and hexagonal plane groups we have used a hexagon as the elementary polygon. The figure in brackets shows the number of times an elementary rectangle is found to recur. The table shows that the frequency of plane groups (the number of varieties in which they occur) decreases as their symmetry rises (the multiplicity of points with weight equal to 1).

Table 3

Plane group	Number of varieties	Recurrence of points with weight = 1	Plane group	Number of varieties	Recurrence of points with weight = 1
C_1^I	713	1	C_{2v}^{IV}	21	4(8)
C_s^I	189	2	C_4^I	18	4
C_s^{II}	124	2	C_{3v}^I	10	6(12)
C_2^I	123	2	C_{3v}^{II}	10	6(12)
C_3^I	47	3(6)	C_{4v}^I	9	8
C_{2v}^I	36	4	C_{4v}^{II}	5	8
C_{2v}^{III}	36	4	C_6^I	5	6(12)
C_s^{III}	27	4	C_{6v}^I	2	12(24)
C_{2v}^{II}	27	4			

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REFERENCES

[1] G. B. Boky, C. R. Acad. Sci. URSS, XXI, No. 5 (1938). [2] Г. Б. Бокий, Тр. лабор. кристаллографии АН СССР, в. 2, 13 (1940). [3] J. D. Donnay and D. Harker, The American Mineralogist, **22**, No. 5, 446 (1937). [4] Internationale Tabellen zur Bestimmung d. Kristalle, 1935. [5] Е. С. Федоров, Записки Минералог. об-ва, 2 сер., XXVIII, 345 (1891). [6] G. Polya, Z. f. Kristallographie, **60**, 278 (1924). [7] P. Niggli, ibid., **60**, 283 (1924). [8] А. В. Шубников, Закон симметрии и кристаллохимия (в книге Гассель «Кристаллохимия», 1936).

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OCEANOLOGY

FORMATION OF ACCUMULATIVE FORMS ALONG WITH CHANGES IN THE DIRECTION OF THE FUNDAMENTAL SHORELINE

By V. P. ZENKOVICH

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Decrease in velocity of the longitudinal coastal shifting of debris is one of the causes for the formation of accumulative forms of sea and lake shores. The determinative rôle, moreover, is played by the shifting of the debris in the upper part of the underwater slope and in the zone between the surf line and the shore proper. The velocity of shifting of the debris along the coastline or along any isobat (ρ) can be expressed by the following equation: $\rho = kE \cos \alpha$, where E is the wave energy; α, the angle of approach of the wave to the shore; and k, a coefficient whose value depends on the depth at the place under consideration, the particle size of the debris, the declivity of the bottom and the nature of the disturbance (1).

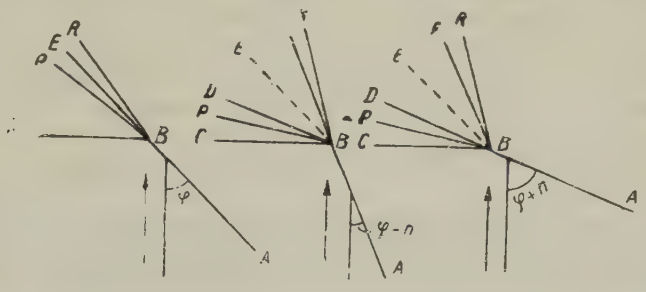
When in the open sea a wave spreads along a normal to the shore, $\cos \alpha$ equals zero, and there is, therefore, no shifting of debris. If the wave goes parallel to the shore, then $\cos \alpha$ is at its maximum for any horizon. At the same time, however, the energy of the wave decreases considerably. This loss of energy is due, first of all, to the elongation of the path of the wave over the shoal water (2), and, secondly, to the fact that the wave, bending under the influence of refraction, expands its front. Moreover, the greater the bend the wave has undergone, the larger the stretch of shore corresponding to the quantity of energy in a given length of wave in the open sea.

It may be considered, approximately, that the loss in energy of the wave is in proportion to the distance from the base of the underwater slope to a point on the shore, lying on the extension of the line of spreading of the wave in the open sea. Then it can easily be shown, as was done by de Lamblardie as far back as in 1789 (3), that the velocity of coastal shifting of debris will be at its maximum when the waves in the open sea spread at an angle of 45° to the shore.

Shoulejkin's theory of wave refraction(4) makes it possible to compute the direction and the length of the wave's ray, but the problem of the energy loss has not been developed sufficiently for us to be able to determine its magnitude for various directions of the wave in the open sea. It is clear from the above, however, that with a certain angle φ the velocity of shifting of the debris along the coastline is at its maximum. φ is not a constant value, but fluctuates within certain limits (approximately 45—30°), depending on the particle size of the debris, the nature of the waves and the declivity of the bottom.

Thus, a change in the direction of the coastline may stipulate a decrease in φ in two ways, and so result in the formation of the two kinds of elementary* accumulative forms described below.

If the mean resultant of the wave in some section is directed towards the shore at an acute angle, a debris stream will form in that section. Since the force and direction of the disturbance is always changing, the stream may have opposite



directions and unequal velocities at various periods. The algebraic sum of these separate motions determines the total shift of the debris during a given period of time, and its mean velocity. The place of origin of the stream may be a section of abrasion shore or the mouth of a river (or rivulet).

A stream has the following characteristics:

1. Capacity, the quantity of material the waves are capable of shifting through a given section of the shore in a given length of time. All other conditions equal, the capacity is in proportion to the velocity of shift.

2. Strength, the quantity of material really shifted by the waves.

3. Saturation degree, the stream is saturated if the shore and the underwater slope are entirely covered with debris, and the waves get all the material they are able to shift. In this case the wave energy is expended only on the shifting of the debris. If, on the contrary, there are bare sections of primary rock on the bottom and on the shore, the load will decrease in proportion to the size of these sections. As a result of this, the stream will not be saturated, and part of the energy of the waves will be expended on the erosion of the shore and the bottom.

A decrease in the capacity of a saturated stream results in the deposition of debris along the shore and the underwater slope. If the stream is not saturated, the saturation degree first rises to full saturation, and only after this deposition sets in.

If the fundamental shore line AB, along which a saturated debris stream is moving, is at an angle φ to the resultant of wave motions (see figure, on which arrows show the direction of the disturbance resultant), then if the shore deviates in either direction, the velocity of shift decreases. If it deviates seawards, a re-entering angle is formed, which fills up with debris. Owing to loss in velocity, the deposition of debris in each growth phase of the form theoretically continues along the fundamental shoreline to the right, up to the source of the material. The direction of the outer line of the accumulative shore during the formation of the form is determined by the ratio of shift velocities along the new and old (AB) segments of shoreline. If the second arm of the re-entering angle is a radius CB along which no debris shifting takes place, the outer shore of the form will become parallel to AB.

Sector EBC may be called the filling sector. The radius CB is at right angles to the direction of the waves. If the angle between them be greater, a contrary debris stream will arise, and the accumulative form will be complex, since it is made up of two elementary ones.

* We term elementary those accumulative forms which arise and develop under the action of a single factor, to distinguish them from complex ones.

If the shore deviates landwards, a point (cape) will form. Since the velocity of shift of the debris on the other side of it is lower than on the segment AB, a mass of debris will accumulate immediately behind point B. The coming of ever new portions will make the accumulative form tear away from the shore and draw out into a spit in the direction of shift of the saturated stream of given strength, in our case, along the extension of the fundamental shoreline AB. Growth of the spit to the left or to the right of this direction is impossible. In the first case the erosion velocity of the accumulative body would be greater than the velocity of feed of the material. In the second, the velocity of feed would be greater than the erosion velocity. Only if the conditions are such that the amount of material carried away from each point of the spit is exactly equal to the amount carried in, will the form be stable.

Sector EBA, situated to the right of the radius EB, we call the shifting-around-a-point sector.

If the shore is at an angle greater or less than φ to the resultant, the conditions of growth of accumulative forms undergo a considerable change. Refer to the figure and consider the first case ($\varphi - n$). If the shore changes its direction seawards, the capacity of the stream in this case first increases, reaches its maximum with the angle φ (radius EB) and then decreases again to its former values (radius DB). Therefore, the shore will be eroded in the sector DBF (the erosion sector).

Only if the bend is greater (to the left of DB), will filling of the angle set in. Under our conditions this process will take place differently than with the angle φ, since the line of the growing accumulative shore will immediately assume the direction of φ (parallel to EB). After this the nuclear triangular form will go on growing, remaining always parallel to itself, its length along AB being limited.

In the shifting-around-a-point sector (FBA) we find the conditions analogous to those described, since the spit can continue to grow, just as before, only in the direction of shift of a saturated stream of given strength, following the line AB.

When the fundamental shore is at an angle greater than $\varphi'(\varphi + n)$, the erosion sector forms where the shore is interrupted landwards. Hence it follows that in this case the spit will always be bent landwards, and the greater the value of n, the greater the angle of the bend (DBF).

If the debris stream flowing along the fundamental shore line is unsaturated, then along a sector of definite width (depending on the deficit) decrease in capacity will not result in deposition. In all three schemes it will begin at the outer side of the radii PB and RB, where the stream becomes fully saturated.

If, on the other hand, the velocity of shift and the capacity in this case increase, then the erosion which accompanied the shifting of the unsaturated stream along the primary shore AB must also increase. Therefore, we must include in the diagrams we have considered the saturation sectors (PBD, FBR and PBR), and term the erosion sector (DBF) in this case the increased erosion sector. Accumulative coastal forms begin to grow only at the outer boundaries of the saturation sectors.

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17.VII.1944.

REFERENCES

[1] И. Мунх-Петерсен, IV Гидрол. конф. Балтийских стран, III, Л., 1933. [2] В. Г. Андреянов, Изв. п.-п. ин-та гидротехники, Л., **25** (1939). [3] De Lamblardie, Mémoire sur les côtes de la haute Normandie, Havre, 1789. [4] W. W. Shoulejkin, Bull. Acad. Sci. URSS, sér. phys.-math., No. 10 (1935).

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BIOPHYSICS

ABSORPTION SPECTRUM OF HAEMOGLOBIN IN SOLUTION AND IN A SUSPENSION OF RED BLOOD CORPUSCLES

By A. A. ILYINA, Kh. M. RAVIKOVICH, D. L. RUBINSTEIN and E. V. SHPOLSKY

(Communicated by S. I. Vavilov, Member of the Academy, 28. XII. 1944)

The absorption spectrum of oxyhaemoglobin is characterized by three bands, two (α and β) in the yellow-green region, and one, known as the γ or Soret band, in the violet. The latter, the broadest and most intense of the three, has a maximum at ~410—414 mμ and is characteristic not only of the closest derivatives of haemoglobin (with variations in the position of the maximum from 405 to 430 mμ), but also of all compounds having a closed tetrapyrolic nucleus.

Macallum and Bradley (1) demonstrated in 1930 that the γ-band is absent in the haemoglobin of intact red blood corpuscles, whereas the two haemoglobin bands in the visible part of the spectrum remain unchanged. This striking phenomenon was confirmed by Adams, Bradley and Macallum (2), Adams(3) and, more recently, by Keilin and Hartree(4). It can easily be reproduced with the aid of the usual spectroscopic technique if a suspension of red blood corpuscles is studied in physiological saline. It can also be observed visually with the aid of the simple methods described by Keilin(4).

Experiments undertaken with a view to determine the nature of this phenomenon failed to give consistent results. Adams showed(3) that a change in the volume of the red cells, produced by placing them in solutions of a NaCl concentration varying from 0.7 to 1.2 per cent does not cause the γ-band to reappear. The spectrum of free haemoglobin, however, does not change even if the latter precipitates from aqueous solution, forming a highly disperse stable suspension. On these grounds Adams excluded scattering of light as a factor responsible for the disappearance of the γ-band and made an attempt to account for this phenomenon by postulating a chemical change of the haemoglobin inside the red corpuscles. He supported this interpretation by model experiments in which haemoglobin was allowed to form a labile compound with stromatin (the structural protein of the red cell stroma). According to Adams, the absorption spectrum of this compound was also devoid of the γ-band. Keilin and Hartree who reproduced these experiments did not confirm Adams' finding as to the obliteration of the γ-band upon interaction between haemoglobin and stromatin. They showed, however, that the γ-band vanishes if a concentrated solution of haemoglobin is emulsified in paraffin or castor oil.

Red cell suspension is a turbid medium. It is a well known fact that the measurement of absorption spectra of pigments in turbid media involves important difficulties and may lead to erroneous results. A simple method was devised by us which made it possible to investigate absorption spectra in turbid media with-

out losses of scattered light or resorting to «standard» turbid media for comparison. The method was based on a special form of polar diagram (indicatrix) of light scattered by large particles, which is characterized by asymmetry with a great preponderance of scattering ahead of the ray ([5]). This asymmetry may be effectively utilized in a photoelectric spectrophotometer with a barrier-layer photocell on account of the plane sensitive surface of the latter.

When investigating transparent objects, the cuvette containing the solution is placed in the path of the beam of parallel rays transmitted by the monochro-

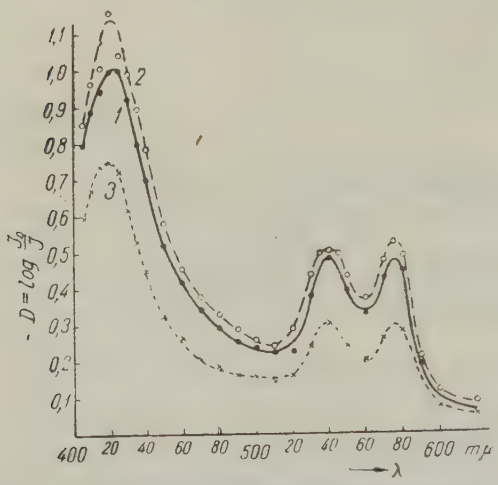

Fig. 1. Absorption spectra of oxyhaemoglobin solutions and red cell suspension. Thickness of cuvette, 1.04 mm. *1*, solution of oxyhaemoglobin; *2*, red cells in physiological saline (1:100); *3*, stabilized fresh blood diluted with saline (1:200)

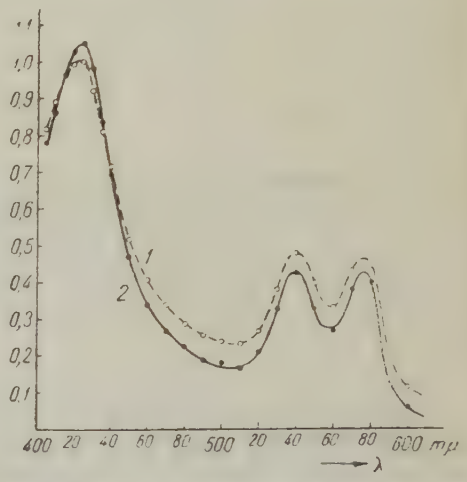

Fig. 2. Absorption spectra of blood before and after haemolysis with saponin. Thickness of cuvette, 5 mm. *1*, red cell suspension in saline; *2*, same, after haemolysis

mator. The rays that leave the cuvette are focussed on the sensitive surface of the photocell. With turbid objects, however, incorrect results are obtained, since the degree of the reduction of light intensity depends upon the position of the cuvette. In order to measure the absorption in turbid media the object under investigation was, therefore, brought in direct contact with the sensitive surface of the photocell. By adjusting the position of the lens a sharp image of the exit slit is obtained on the front surface of the cuvette, forming a very narrow band approximately 0.05 mm wide. Upon passing through the turbid medium the light is scattered and gives a diffuse image, all of which, however, is projected upon the sensitive area of the photocell. Under such conditions nearly all the light that has passed through the object under investigation, except for a small fraction scattered in the opposite direction, falls on the recording instrument so that a turbid standard can be dispensed with.

In the set-up used by the authors a selenium photocell was employed, manufactured at the State Optical Institute. The photoelectric current was measured with a galvanometer sensitive to 10^{-9} A/mm at a scale distance of 2.5 m, period 0.2 sec. A plane parallel quartz cuvette, 1.04 to 2.5 mm thick, filled with a diluted suspension of red blood corpuscles, was attached directly to the photocell surface. The constancy of the incident light intensity was checked by another photocell connected to a second galvanometer, and was rigidly controlled throughout each experiment.

Suspensions of red cells in physiological saline were prepared for investigation by diluting with a 0.9 per cent NaCl solution either fresh blood taken from donors, or centrifuged and thoroughly washed human red blood corpuscles. A solution of oxyhaemoglobin prepared from red cells served for comparison. The absorption curves reproduced in Fig. 1 show the γ-band with a maximum at ~420 mμ

both in a solution of oxyhaemoglobin and in a suspension of red corpuscles. In the latter the γ-band shows no tendency to vanish. The experiment pictured in Fig. 2 is especially convincing. In this case red blood cells were subjected to saponin haemolysis directly after their absorption spectrum had been recorded; another spectrum was taken of the haemolyzed blood. As may be seen from Fig. 2, the two absorption curves nearly coincide.

The slight displacement of the maximum of the γ-band towards the long-wave part of the spectrum, as observed in this experiment (420 mμ instead of the normal position of the maximum at 410—414 mμ), can be accounted for by the wide entrance slit of the monochromator, which had to be used on account of

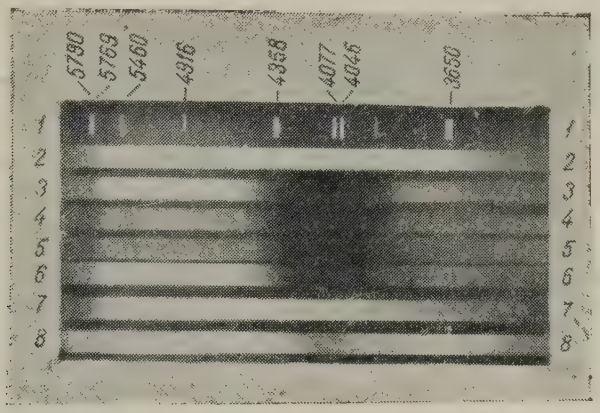

Fig. 3. *1*, spectrum of mercury arc; *2*, spectrum of incandescent lamp scattered by white paper at an angle of 45°; *3*, spectrum of light scattered by haemolyzed blood; same illumination at same angle behind the cuvette (d=2.5 mm), with paper; *4*, same, with ground glass behind the cuvette; *5*—*8*, spectra of light scattered by red cell suspension in same cuvette; *5*, illumination at an angle of 45°; no substratum behind the cuvette; *6*, white paper behind the cuvette (cf. *3*); *7*, in transmitted light; *8*, with light source shifted from the axis of the spectrograph (an angle of 135° with the transmitted ray)

the low sensitivity of our set-up in the short-wave part of the spectrum. The normal position of the γ-band maximum at 410 mμ was obtained in additional measurements made with another, highly sensitive spectrophotometer unit, comprising an antimony-caesium photocell and an electrometer as a recording instrument.

A very narrow slit (0.05 mm) could be used in these experiments.

It should be noted that the coincidence of the two curves of Fig. 2, although fairly good, is still not ideal. In the absorption spectrum of the red cells the γ-band, as compared to the α- and β-bands, is found to be somewhat weaker than in the spectrum of free oxyhaemoglobin. This slight dimming of the γ-band seems to be a faint manifestation of the light scattering effect which in its extreme manifestation leads to the total obliteration of this band described by the British writers. This finds support in the experiments with the Sb—Cs photocell mentioned above. Here the spherical bulb of the photocell prevents a direct contact between the cuvette and the sensitive surface of the photocell. As a result the scattering effect is enhanced and the discrepancy between the light absorption curve of the red cells and that of free haemoglobin in the region of the γ-band becomes quite significant. When the absorption spectrum of red cells is studied with the aid of such a photocell, the γ-band, although clearly discernible, is appreciably weaker in the spectrum of free haemoglobin.

Numerous replicate experiments in which the slightest evidence of haemolysis was carefully avoided, make it impossible to ascribe the results obtained to any injury of the corpuscles. Thus, by applying the above photoelectric method to the study of intact red blood corpuscles, a complete absorption spectrum of

intraglobular oxyhaemoglobin can be obtained, including the γ-band which defied the routine spectrographic methods used by the earlier students.

In order to correlate the results obtained with those of the previous investigators, the following experiments were carried out. A cuvette containing either a suspension of red cells or a solution of haemoglobin was mounted on the optical bench of a quartz spectrograph (a Baush or a Lomb «Medium» quartz spectrograph).

In different experiments the light source was set up in the following three positions: 1) sidewise of the spectrograph slit, so as to obtain an angle of 45° between the light beam and the axis of the bench; 2) so that the light beam formed an angle of 135° with the optical axis of the bench; 3) on the optical axis of the spectrograph (the usual position, photography of the absorption spectrum in transmitted light).

The spectrograms obtained are presented in Fig. 3. It will be seen that the γ-band disappears when the light source is in position *3*. With the light source in position *1* the band is as intense as in haemolyzed blood; and, finally, with position *2* the band, although present, is much reduced in intensity.

These data conclusively show that the eclipsing of the γ-band in the absorption spectrum of red blood corpuscles described by the British authors is a peculiar optical phenomenon dependent on the scattering of light by large dispersed particles. Our methods of spectrum analysis of scattered light make it possible to detect the obliterated γ-band in a suspension of intact red cells. The study of the nature of this optical phenomenon is in progress. But even now it is obvious that intraglobular haemoglobin retains all its absorption bands and does not differ from free haemoglobin in its light absorbing properties.

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REFERENCES

[1] A. B. Macallum and R. C. Bradley, Science, **71**, 341 (1930). [2] G. A. Adams, R. C. Bradley and A. B. Macallum, Biochem. J., **28**, 482 (1934). [3] G. A. Adams, ibid., **32**, 646 (1938). [4] D. Keilin and E. F. Hartree, Nature, July, 19, No. 3742, 75 (1941). [5] И. А. Хвостиков, Успехи физич. наук, **24**, 165 (1940).

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CHEMISTRY

FUSION DIAGRAM FOR THE SYSTEM KF — NaF — MgF_2

By A. G. BERGMAN and E. P. DERGUNOV

(Communicated by I. I. Černjaiev, Member of the Academy, 15. X. 1944)

This system is one of the four side triple systems forming the faces of the composition tetrahedron of a quadruple system of Li, K, Na and Mg fluorides which we are engaged in studying. Chemical reaction is manifested to a very high degree in this system; three compounds are formed in it: $KF \cdot MgF_2$, $2KF \cdot MgF_2$, $NaF_2 \cdot MgF_2$.

According to Remy and Seeman [1], one more compound, $3KF \cdot 2MgF_2$, is formed, but this will be discussed below. We were unable to detect this compound on the crystallization surface of the system.

In systems of the fluorides and chlorides of K and Mg there is a parallelism with two carnallites: $KG \cdot MgG_2$ and $2KG \cdot MgG_2$. But in systems of the chlorides and fluorides of Na and Mg the types of compounds are pronouncedly diverse; the fluorides can form compounds only with a ratio of 1 : 1, and the chlorides, only with ratios of 2 : 1 or 1 : 2.

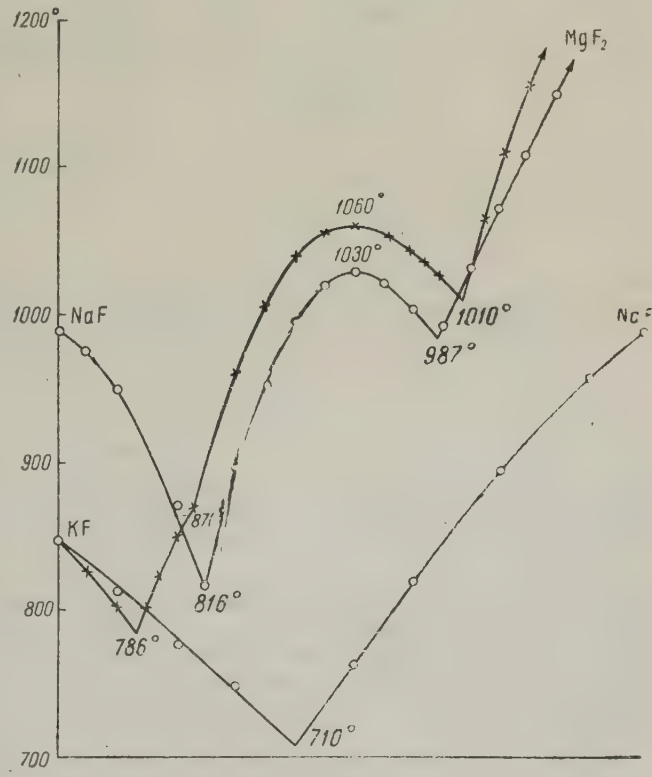

Fig. 1

The compounds in the triple system of K, Na and Mg fluorides, which we are dealing with in this paper, determine the nature of the whole quadruple system, as a whole, because the fourth component LiF does not form any additional compounds and serves only as a solvent.

One of the purposes of this study was to find fluoride fluxes for the refining of non-ferro metals and their alloys.

The method used for the study was the visual-polythermal method. The temperature at which the first crystals appeared and that at which the last of them disappeared, were determined by means of a Pt/Pt, Rh thermocouple whose bare joint was submerged in the melt in a platinum crucible, which was placed in a vertical electric furnace and illuminated from above by means of a projection light.

Double Systems: 1. KF—NaF (Fig. 1) has been studied by Kurnakov and Zhemchouzhny ([2]). Dombrovsky and Koloskova ([3]), Poliakov ([4]) and others.

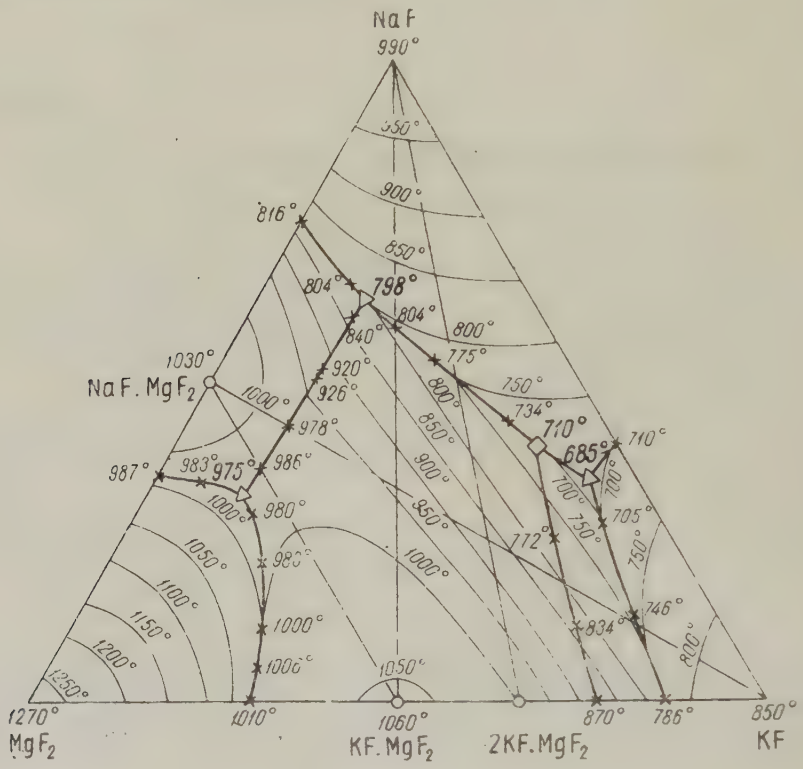

Fig. 2.

According to the data of these authors and to ours, this system forms an eutectic at 710° with 40 per cent (mol.) NAF and with limited quantities of solid solutions ([5]), containing from 0 to 12 per cent NaF and from 97 to 100 per cent NaF.

2. $NaF—MgF_2$ (Fig. 1). Studied by Grube([6]) and repeated again by us. The system forms the compound $NaF \cdot MgF_2$ with an open maximum at 50 per cent MgF_2 and 1030°. The compound $NaF \cdot MgF_2$ forms an eutectic with NaF at 816°, containing 75 per cent NaF, while the eutectic between the compound and MgF_2 is at 987° and 64 per cent MgF_2. Solid solutions have not been observed to form in this system.

3. $KF—MgF_2$ (Fig. 1). Studied by Remy and Seeman ([1]), Bergman and Pavlenko ([7]) and verified by us. On the basis of the prediction developed by Remy and Pelens, concerning the relation between the ionic charges, the ionic radii and the stoichiometric coordinational numbers, it should have been expected that magnesium fluoride forms two compounds with potassium fluoride of the types: $M^+F \cdot MgF_2$ and $2M^+F \cdot MgF_2$. Remy and Seeman's experimental data and ours have confirmed this prediction.

The compound $KF \cdot MgF_2$ was produced by precipitating from aqueous solution as far back as 1925 by von Arkel ([8]), who determined its structure roentge-

nographically. Dubois ([9]) introduced MgO into fused KF and then leached out the melt after it had solidified. On the basis of the analytical compositions of the residues he gave them the following formulae: $KF \cdot MgF_2$ and $2KF \cdot MgF_2$. The compound $KF \cdot MgF_2$ melts congruently at 1060° and does not form solid solutions with an excess of MgF_2: the eutectic has a melting point of 1010° and a composition of 68 per cent MgF_2 and 32 per cent KF.

The transition point at 870° and 22 per cent MgF_2 corresponds incongruently to the compound $2KF \cdot MgF_2$ at its melting point. Remy and Seeman consider that the compounds $KF \cdot MgF_2$ and $2KF \cdot MgF_2$ form a third compound $3KF \cdot 2MgF_2$, and that this compound is retained afterwards, when the solid phase cools down. Our experiments failed to establish any possibility of the compound $3KF \cdot 2MgF_2$ forming.

In order to construct the complete liquidus surface, 10 sections have been drawn inside of the equilibrium triangle. These sections have been drawn from the binary sides NaF--KF to MgF_2, from NaF—MgF_2 to KF, and from KF—MgF_2 to NaF. They establish the lines of joint crystallization in the field of compounds and components.

To conclude the investigation, a space diagram of state has been constructed for the system NaF—KF—MgF_2. A general view of this diagram is given in Fig. 2. The composition triangle has been divided by the joint crystallization curves into six fields: $KF \cdot MgF_2$ (39.2 per cent of the total area), $2KF \cdot MgF_2$ (6.7 per cent), MgF_2 (15.5 per cent), $NaF \cdot MgF_2$ (10.1 per cent), NaF (21.0 per cent) and KF (7.5 per cent). The large area occupied in the composition triangle by the field of the compound $KF \cdot MgF_2$ is a point of some interest, seeing that its melting point (1060°) is lower than that of the most refractory component MgF_2 (1270°).

The formation of the three compounds $2KF \cdot MgF_2$, $KF \cdot MgF_2$ and $NaF \cdot MgF_2$ divides the composition triangle up into four simple systems:

1. $KF - 2KF \cdot MgF_2 - NaF$. 2. $2KF \cdot MgF_2 - KF \cdot MgF_2 - NaF$
3. $NaF \cdot MgF_2 - KF - NaF$. 4. $NaF \cdot MgF_2 - KF - MgF_2$

Therefore there are three eutectic and one transition triple points in the system, which correspond to the following compositions and temperatures:

Nature of point	Composition in mol. per cents			Temperature °C	Equilibrium phases
	MgF_2	NaF	KF		
Eutectic	53.5	33.0	13.5	975	MgF_2; $KF \cdot MgF_2$; $NaF \cdot MgF_2$
»	22.5	62.5	15.0	798	$NaF \cdot MgF_2$; $KF \cdot MgF_2$; NaF
»	6.5	34 5	59.0	685	KF; NaG; $2KF \cdot MgF_2$
Transition	11.0	39.0	50.0	710	NaF; $2\ KF \cdot MgF_2$; $KF \cdot MgF_2$

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REFERENCES

[1] H. Remy et W. Seeman, Rec. des Trav. chim. des Pays-Bas, **59**, No. 7/8, 516 (1940). [2] Н. С. Курнаков и С. Ф. Жемчужный, Z. anorg. Chem. **52**, 186 (1907). [3] Н. С. Домбровская и З. А. Колоскова, Изв. сектора физ.-хим. анализа, **10**, 211 (1938). [4] В. Д. Поляков, там же, **11**, 201 (1938). [5] С. Мукимов, там же, **11**, 275 (1938). [6] G. Grube, Z. Elektrochem., **33**, 11, 481 (1927). [7] A. G. Bergman and S. P. Pavlenko, C. R. Acad. Sci. URSS, XXX, No. 9 (1941). [8] A. E. von Arkel, Physika, **5**, 162 (1925). [9] A. Dubois, C. R., **120**, 678 (1895).

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CHEMISTRY

A NEW METHOD OF PRODUCING SPECTRAL-PURE PLATINUM

By I. I. ČERNJAIEV, Member of the Academy,

and A. M. RUBINSTEIN

Modern methods of producing chemically pure platinum have made it possible to obtain platinum with a sufficient degree of purity. In 1928 one of the authors of this paper suggested a method of producing extra-pure platinum through Peyronet's salt $(NH_3Cl)_2Pt$.

When this method had been mastered, it was found that by means of it extra-pure platinum could be produced with a platinum content of 99.9998 per cent. Analysis of this platinum revealed that it contained an admixture of 0.0001 per cent palladium and 0.0001 per cent calcium and sodium. Iridium, rhodium and gold were not discovered. This method consists in producing platinum by precipitating it out of an ammonium chloroplatinite solution as Peyronet's salt $(NH_3Cl)_2Pt$. This salt was selected for the purpose on account of its great purity. Peyronet's salt is a non-electrolyte, has no external sphere, and therefore in crystallizing does not adsorb upon its surface admixtures of salts of the other metals, commonly present in the solution. This method was meant for chloroplatinates containing considerable amounts of iridium and gold.

Now we suggest a new method by which platinum can be prepared in a state of purity even greater than this. This method gives good results even when the palladium content in the platinum is high. We have used it to produce spectral-pure platinum from platinum containing as much as 2 per cent palladium. For this purpose we use Blomstrand's salt $(NH_3NO_2)_2Cl_2Pt$. The essential points of the method are as follows: metallic or spongy platinum is dissolved in aqua regia (one part of nitric to three parts of hydrochloric acid). The resulting solution is evaporated twice with hydrochloric acid and once with hot water. This operation must be carried out with great care in order to drive off the nitric acid as completely as possible, and to decompose all the nitroso-compounds which form with the platinum. The resulting solution of H_2PtCl_6 is diluted with water in sufficient quantity to make the platinum concentration not more than 100 g per litre. The diluted H_2PtCl_6 solution is treated with a 10 per cent solution of potassium chloride. The resulting yellow precipitate of potassium chloroplatinate is rapidly filtered, washed with water and alcohol and dried. After this the chloroplatinate is put through a nitrite treatment, for which purpose it is heated in aqueous solution with potassium nitrite. The quantity of KNO_2 taken is such as to total about 8 molecules KNO_2 per molecule of K_2PtCl_6. The reaction is complete when no more oxides of nitrogen can be liberated, and the whole solution turns colourless. Besides the reaction of nitrification, when K_2PtCl_6 is treated with KNO_2, platinum is reduced to its bivalent form, giving in solution the compound $K_2Pt(NO_2)_4$. After filtering, the platinonitrite solution $K_2Pt(NO_2)_4$ is

treated with a 20 per cent ammonia solution. This results in the immediate appearance of an abundant white crystalline precipitate of the cis-configuration of diaminodinitroplatinum $(NH_3NO_2)_2Pt$. The precipitate is filtered and washed with cold water. Precipitating platinum as the diaminodinitrite results in a salt which adsorbs only very small quantities of admixtures from the solution. In order to purify this salt further, it is necessary to produce some derivative of it, which would be soluble in water and so could be recrystallized.

We selected the method of oxidizing the salt with gaseous chlorine to form Blomstrands' salt according to the reaction:

$$(NH_3NO_2)_2\,Pt + Cl_2 \rightarrow (NH_3NN_2)_2\,Cl_2\,Pt$$

The diaminodinitrite precipitate was suspended in water, and chlorine was bubbled through in the cold. This resulted in the precipitate being partly dissolved and partly oxidized in the solid state. Chlorine must be bubbled through the solution until a sample observed under the microscope shows the presence of only one phase, and the colouring of the solution will not disappear when shaken in the cold. The resulting Blomstrand's salt is crystallized from water twice or three times. It should be noted that the yield of this salt decreases upon recrystallization from water. During this process part of the salt is transformed to some other form giving viscous solutions from which Blomstrand's salt cannot be liberated. At present one of us is engaged in an elaborate study of Blomstrand's salt with the purpose of explaining the chemical nature of these solutions.

Great care must be exercised when calcining Blomstrand's salt to produce the metallic platinum. If it is heated too rapidly, the salt may explode and result in a loss of platinum.

The pure platinum we produced by this method was put to spectral analysis. The results of this analysis showed that the platinum is spectral-pure, and contains absolutely no iridium, palladium nor iron. The physical properties of this platinum were investigated at the Institute of Physical Problems of the Academy of Sciences of the USSR, proving it to be absolutely pure.

This method can be used on a laboratory scale to produce spectral-pure platinum, being simpler and more reliable than any other method described in literature.

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GEOCHEMISTRY

METAMORPHIZATION OF WATER OF LAKE BALKHASH

By V. D. KONSHIN

(Communicated by D. S. Beliankin, Member of the Academy, 13. IX. 1944)

A party sent by the Institute of Geological Sciences of the Academy of Sciences of the USSR, under the guidance of D. G. Sapozhnikov, head of the party, and N. M. Strakhov, was at work on lake Balkhash in 1941. One of the objects of the work was to determine the hydrochemical conditions of formation of dolomite in the lake. The dolomite was discovered in the bottom deposits by the same party in 1940 (1). Without dwelling upon this problem as a whole, we give here some data concerning the metamorphization of the water of lake Balkhash.

The specimens of water were taken from eight stations situated along the length of the lake and plotted on the accompanying schematic map. The four stations in the western part recorded during the period from July 7 to July 18; the other four stations, in the eastern part, during the period from August 19 to September 9. Such an interval between the terms of sampling can possibly partly account for the striking difference in the data for the eastern and the western part of the lake.

The determination of carbonates and bicarbonates was made by titration with HCl in the presence of phenolphtalein and methyl-orange. The sulphates were determined by the benzidine method (in Zaprometov's modification); chlorides, after Mohr; calcium and magnesium, by the gravimetrical methods. The total of alkaline metals was calculated as the difference of milligram-equivalents of anions and cations, with a subsequent recalculation to weight amount (the K : Na ratio has been taken as constant for the whole lake and equal to 1 : 9). Total salinity was calculated as the sum of all the ions. pH in the western part of the lake was determined according to Gillespi; in the eastern part, according to the Clarke-Palich scale.

Lake Balkhash is known to be located in the eastern part of Central Kazakhstan, in a country with a sharply continental arid climate. Having no drainage the water supplied to the lake is gradually evaporated, and its salinity increases gradually.

The length of the lake (about 600 km) being 10 times its width (not above 60 km), the peculiarities of the hydrological regime of lake Balkhash consist in that up to 80 per cent(2) of the water is supplied to it from the Ili River, discharging into the south-western part of the lake. Consequently, it is there that the water shows the lowest salt concentration (1261 mg/l, Table 1). Further eastward, as a result of evaporation, the water gradually becomes higher in salts, to attain the highest salinity in the extreme eastern reach (5243 mg/l). The salinity is found to increase more than four times.

Table 1

Ionic Composition of Water of Lake Balkhash, mg/l

Station	CO_3''	HCO_3'	SO_4''	Cl'	$Ca^{\cdot\cdot}$	$Mg^{\cdot\cdot}$	$Na^{\cdot}+K^{\cdot}$	Total
112	20.5	280.5	354	228	57.4	74.0	267	1261
110	22.8	270.9	370	237	55.7	77.0	252	1285
120	22.2	292.1	395	260	37.4	81.9	308	1389
128	25.2	334.3	486	317	42.2	99	363	1667
164	48.9	443.8	893	574	25.7	164	694	2843
150	86.6	628.4	1410	954	17.0	240	1174	4510
138	102.8	683.8	1628	1081	15.1	285	1315	5110
135	122.1	662.8	1685	1107	14.0	289	1363	5243

However, as it will be seen from the analysis, the water of the eastern part of lake Balkhash is not identical with that of the western part of the lake with regard to its salt composition. Largely under the influence of concentration,

Schematic map of lake Balkhash

definite salts are precipitated to the bottom, as a result of which the water becomes relatively enriched in others. Even if we are to express the data of analysis in absolute figures (in mg/l, Table 1), along with an increase in the concentration of all the ions we observe a decrease in calcium content. Much more demonstrative are the relative figures in per cent to the total salinity, particularly with respect to chlorine (the so-called chlorine coefficients, Table 2), since chlorine does not play any perceptible part in chemical adsorption or in biological processes taking place in lakes (except for the precipitation of sodium salts in strongly concentrated salt lakes), and can therefore serve as an index of increasing concentration.

Table 2

Chlorine Coefficients of the Water of Lake Balkhash

Station	CO_3''	HCO_3'	SO_4''	$Ca^{\cdot\cdot}$	$Mg^{\cdot\cdot}$	$Na^{\cdot}+K^{\cdot}$	Total
112	9.00	123.02	155.26	25.17	32.46	117.11	5.53
110	9.62	114.30	156.12	23.50	32.49	106.33	5.42
120	8.54	112.35	151.92	14.38	31.50	115.38	5.34
128	7.95	105.46	153.31	13.31	31.23	114.51	5.26
164	8.52	77.32	155.57	4.48	28.57	120.91	4.95
150	9.08	65.87	147.80	1.78	25.16	123.06	4.73
138	9.51	63.26	150.60	1.40	26.36	121.65	4.73
135	11.03	59.87	152.21	1.26	26.11	123.13	4.74

The chlorine coefficient for the total of salts is found somewhat to decrease from West to East, thus pointing to a passage of some of the component salts of the water into precipitate. If this process were not operative here, the concentration of the salts would attain 6122 mg/l.

As to individual ions, the chlorine coefficients show an increase from West to East for alkaline metals only, while for sulphates and carbonates they remain almost unchanged. A certain decrease is observed in the eastern part of the lake for magnesium. A considerable decrease is manifest in bicarbonates (twice) and particularly in calcium (eighteen times). All this may easily be related to the formation of calcareous and calcareo-dolomitic deposits in the lake.

Table 3

Salt Composition (mg/l) and pH of the Water of Lake Balkhash

Station	$CaCO_3$	$Ca(HCO_3)_2$	$MgCO_3$	$Mg(HCO_3)_2$	$MgSO_4$	Na_2SO_4	NaCl+ +KCl	$\frac{Na_2SO_4}{MgSO_4}$	pH
112	34.2	95.9	—	249.9	160.9	333.6	387.2	2.07	8.3
110	38.0	163.8	—	177.0	235.6	269.2	402.9	1.14	8.2
120	37 0	91.4	—	267.8	185.1	365.7	442.6	1.98	8.4
128	42.0	102.8	—	308.2	236.6	439.5	537.9	1.86	8.4
164	64.2	—	14.6	532.3	353.0	904.0	976.0	2.56	9.0
150	42.5	—	85.9	753.7	445.0	1560.0	1625.7	3.51	9.15
138	37.7	—	112.7	820.1	575.2	1728.8	1836.6	3.01	9.2
135	35.0	—	142.1	794.9	573.8	1814.7	1884.5	3.16	9.15

A recalculation of the ions to salts (Table 3) reveals a considerable stability of $CaCO_3$ (save for station 164). The increase in carbonates in the eastern part of the lake leads to the precipitation of calcium and the appearance of $MgCO_3$ in the solution. The latter fact, along with the increase in the concentration of magnesium bicarbonate (4.5 times), leads to the increase in the alkalinity and pH of the water in the eastern part of the lake ([5]).

As may be seen from its salt composition, the water of lake Balkhash contains neither $CaCl_2$, nor $MgCl_2$, but does contain Na_2SO_4. It may therefore be referred to the sulphate type ([4]). The process of metamorphization of such a water is characterized by the following coefficient: $k = Na_2SO_4/MgSO_4$ (Table 3). This coefficient does not express any regular changes from West to East, although in the eastern part its values are much higher. This phenomenon is possibly also related to the process of dolomite formation in the eastern part of the lake.

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13.IX.1944.

REFERENCES

[1] D. G. Sapozhnikov, C. R. Acad. Sci. URSS, XXXVI, No. 4—5 (1942). [2] А. В. Шнитников. Труды Соляной лаборатории АН СССР, XI, Балхаш и соляные озера его бассейна, 1936. [3] В. К. Радищев, Материалы по гидрохимии соляных озер Каспийского бассейна, 1, К гидрохимии озера Эльтон, Работы Волжской биолог. станции, XI, № 1, 1930. [4] Н. С. Курнаков и др., Соляные озера Крыма, 1936.

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BIOCHEMISTRY

CHEMICAL TRANSFORMATIONS AS A SOURCE OF ENERGY FOR THE MOBILITY OF *TRYPANOSOMA EQUIPERDUM*

By I. I. IVANOV and M. V. UMANSKAYA

(Communicated by K. I. Skrjabin, Member of the Academy, 10. IX. 1944)

Kudicke and Evers (1) showed as far back as in 1924 that *Trypanosoma brucei* becomes soon deprived of its mobility when transferred to various saline solutions; these trypanosomes are found to retain their life activity, however, in solutions containing glucose, fructose, mannose, galactose or maltose, as well as in glycerine solutions. The authors cited above did not exclude the possibility that the trypanosomes may directly consume the substances enumerated above as nutrient material and as source of energy; at the same time they alluded, however, to the power of these compounds to eliminate the toxic effect of saline solutions.

This problem received no definitive solution in the study of Reiner, Smythe and Pedlow (2) who made but a transient remark to the effect that when sugars are assimilated by the trypanosomes, the energy liberated during this process is probably used up by those blood-suckers to maintain their mobility.

Ivanov and Yakovlev have shown that trypanosomes soon lose their mobility and finally succumb not because purely saline solutions are a poorly appropriate medium for the life activity of those microorganisms by virtue of a specific toxicity they are presumed to possess, but merely because the solutions tested are lacking certain substances which are indispensable for their nutrition.

These authors have developed a method for rapidly immobilizing trypanosomes in purely saline solutions; they have also demonstrated the possibility to «revive» them by way of adding to those solutions definite mono- or disaccharids.

In the study here reported was demonstrated the possibility to restore the mobility of *Trypanosoma equiperdum* in purely saline media upon addition to a motionless suspension of the microorganisms of pyruvic acid or glycerine (see table). It is of interest to note that by adding cyanide to the solutions under test, the trypanosomes are not at all prevented from consuming glycerine and pyruvic acid as energetic material. This should evidently be put down to the circumstance that the respiration in *Trypanosoma equiperdum* is not affected by hydrocyanic acid (4).

Our observations went to demonstrate that the trypanosomes are highly resistant to sodium fluoride as well, whose addition to their media, even at concentrations exceeding 0.02 mol., does not reduce their mobility under aerobic conditions, provided the media tested contain some of the nutrient substrata mentioned above. It was not until the concentration of NaF was brought to 0.1 mol. that the trypanosomes were found gradually to lose their mobility. But even then, by adding some pyruvic acid, they could not infrequently be restored to motion

Composition of media	Mobility of *Trypanosoma equiperdum* Time after the beginning of experiment in minutes							
	0	10	20	30	40	50	60	70
NaCl phosphor. solution	—	—	—	—	—	—	—	—
CaCl phosphor. solution + 0.3% glycerine (or pyruvate)	—	+	+++	+++	+++	+++	+++	+++
NaCl phosphor. solution+0.3% glycerine+ +NaCN (0.001 *m*) . .	—	+	+++	+++	+++	+++	+++	+++

Note. —, no motion; +, slight mobility; +++, very intense mobility

Thus, as has already been pointed out by Ivanov and Yakovlev ([3]), the mobility of *T. equiperdum* is closely linked with the process of assimilation from the external medium of such various nutrient substances, as sugar, glycerine, pyruvic acid and possibly some other products of carbohydrate exchange.

The trypanosomes do not utilize as energetic material to any appreciable extent their intracellular substrata, either under aerobic or anaerobic conditions.

Here *T. equiperdum* differs sharply from animal spermatozoa. This difference is evidently due to the specific conditions of the life activity of the trypanosomes, mobile microorganisms parasitic of blood plasma which always contains sufficient quantities of nutrient material, whereas the spermatozoa have to make their way to the egg cell through the sex channel of the female, largely consuming the energy of their proper endogenic nutrient resources.

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REFERENCES

[1] R. Kudicke u. E. Evers, Z. Hyg. und Infekt., **101**, 317 (1924). [2] L. Reiner, C. Smythe and J. Pedlow, J. biol. Chem., **113**, 75 (1936). [3] И. И. Иванов и В. Г. Яковлев, Биохимия, **8**, 229 (1943). [4] B. Fenyvessy u. L. Reiner, Z. Hyg. u. Infekt., **102**, 109 (1924).

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CHEMICAL TECHNOLOGY

NATURE OF ACTIVE CENTRES OF ALUMOSILICATE CATALYSTS

By V. M. GRIAZNOV, V. V. KOROBOV and A. V. FROST

(Communicated by V. M. Rodionov, Member of the Academy, 24.X.1944)

The purpose of this investigation was to study the composition of the active complex of alumosilicate catalysts, using the methods, suggested by N. I. Kobozev's ensemble theory(1). It has been established that the indispensable ingredients of alumosilicate catalysts are aluminium oxide and silicon dioxide. The silica gel was found to be inactive, no matter what its origin, for the cracking of gasoil and the enrichening of benzene. The pure alumo gel, on the contrary, was shown by K. V. Topchieva * to be appreciably active. Hence it could be concluded that the silica gel in the catalysts of the reactions mentioned is the bearer, while the activity is determined by the aluminium oxide. The aluminium oxide was deposited onto the silica gel by impregnating the latter with an aqueous solution of aluminium sulphate, acidified with sulphuric acid to a pH of 0.65, in view of the fact that Jander and Jahr (2) have shown that under such conditions the aluminium sulphate is completely dissociated, while a comparison of the data of Denham(3) and Bjerrum (4) shows that the least hydrolysed of all the aluminium salts investigated is the sulphate.

The catalysts were prepared from technical silica gels produced by the Voskresensk Chemical Works. The impurity content, which in the initial gels was as high as 1.49 per cent, was reduced to 0.06 per cent by washing with hydrochloric acid. After being dried in a drying oven, the silica gel was roasted in a muffle oven for two hours at 520° C to condense its structure. After cooling, the silica gel was kept submerged in a solution of aluminium sulphate for 22 hours, this being the least time of soaking that gives maximum activity of the catalyst (soaking for longer than 22 hours does not increase the activity any more). After the impregnating solution had been drawn off by means of a glass filter, the catalyst was washed twice in dilute sulphuric acid with a pH of 0.65 for periods of one half hour. This procedure was necessary in order to remove the excess of the impregnating solution, which otherwise might crystallize out on the surface of the catalyst upon heating. The use of pure water in washing off the excess of the impregnating solution is to be avoided, since this would result in the precipitation of indefinite hydrolysis products. After being washed with acid, the catalyst was rinsed with distilled water to a negative reaction for the sulphate ion in the wash water.

Seven samples of catalysts were prepared, in which the aluminium oxide content was found to be a linear function of the normality of the impregnating solu-

* Unpublished data, produced in the Laboratory of Kinetics and Catalysis of the Lomonosov State University, Moscow.

tions (Table 4). The only exception was catalyst No. 7 impregnated with an almost saturated 12 *N* solution, out of which aluminium sulphate might have crystallized into the larger pores of the surface of the silica gel, inaccessible for the impregnating solution.

The activity of each catalyst was tested in the enrichening of cracking-benzene and in the cracking of gasoil.

1. Enrichening of Benzene. a) Materials—benzene of the 100—150° fraction, distilled out of commercial automobile cracking-benzene in Godaskin retorts. The group composition of the fraction is given in Table 1. b) Conditions of experiments: temperature, 400°C; time of work between regenerations, 1 hour; velocity, 0.6 litres per litre of catalyst per hour.

Table 1

Group Composition of the 100—150° Fraction

Sum of aromatic and unsatur. hydrocarbons, %	Iodine number	Molecular weight	Aniline point	Per cent by weight			
				Unsaturated	Aromatic	Naphthenic	Paraffin
45	78	112	62	35	12	22	31

2. Cracking of Gasoil. a) Materials—the gasoil fraction with a boiling point of 250°C and up, distilled from heavy kerosene produced at the Vladimir Plant. The group composition of the fraction is given in Table 2. b) Conditions of experiments: temperature, 450°; time of work between regenerations, 1 hour; velocity, 0.6 litres per litre of catalyst per hour. The product of the experiment was weighed, and its yield in respect to the initial material was determined.

Table 2

Group Composition of the Kerosene Distillate

Fraction	Yield %	Before sulphidization				After sulphidization			Composition (per cent by weight)				
		d_4^{20}	Unsat.+aromatic	Iodine number	n_D^{20}	Aniline point	n_D^{20}	d_4^{20}	Unsaturated	Aromatic	Naphthenic	Paraffin	Molecular weight
250—300°	50.33	0.8525	20	4.1	1.4740	80.3	1.4585	0.8398	3.0	19.0	39	30	214
300—350°	45.19	0.8609	20	2.6	1.4770	89.2	1.4605	0.8406	3.5	18.5	32	46	242
Residue	4.43	—	—	—	—	—	—	—	—	—	—	—	—

The results of these tests are given in Table 4. Each iodine number and each benzene yield is the mean of two determinations, whose discrepancies were insignificant. In order to verify the reproducibility of the results, two of the catalyst were prepared anew. The results obtained with these catalysts were found to deviate from the former results by a value which lay within the limits of experimental error, or of the error of the method of analysis used. The lack of activity of catalyst No. 7 for cracking can be explained by the assumption made above about the pores of the catalyst being plugged up or their diameter being reduced by the salt crystallizing out of solution, as a result of which the heavy molecules of the gasoil hydrocarbons cannot penetrate into these pores.

The problem as to whether the decrease of the iodine number of benzene could be considered a criterion of the activity of the catalyst in the enrichening reaction, was solved by studying the kinetics of the disproportioning of hydrogen which takes place during the reaction. To this end experiments with varying volume velocities were carried out for some of the catalysts (Table 3).

Table 3

Iodine Numbers of Benzenes as a Function of the Volume Velocity

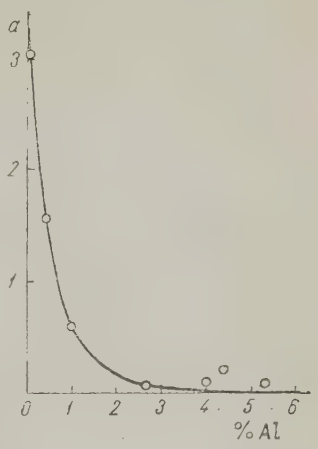
Catalyst	Volume velocity V	Iodine number I_x	$I_0 - I_x$	$k_0 = (I_0 - I_x)V$	$k_1 = V \lg \frac{I_0}{I_x}$
No. 6	0.60	28	50	30.0	0.267
»	0.30	10	68	20.0	0.267
»	0.15	2	76	11.0	0.240
No. 4	0.60	56	22	13.2	0.0864
»	0.45	50	28	12.6	0.0870
»	0.15	21	57	8.5	0.0855

I_0 and I_x are, respectively, the initial and final iodine numbers, which are in proportion with the respective concentrations of unsaturated hydrocarbons; the volume velocity is in inverse proportion with the time of reaction. The reaction velocity constant of the first order $k_1 = V \lg \frac{I_0}{I_x}$ shows the best constancy; hence, it may be concluded that for the reaction of enrichening the activity of the catalyst should be characterized by the logarithm of the ratio of the iodine numbers of the initial fraction and the enrichened benzene: $A = \lg \frac{I_0}{I_x}$.

Hence, the specific activity (assuming $\% \; Al = \alpha$)

$$a = \frac{A}{\% \, Al} = \frac{\lg \frac{I_0}{I_x}}{\% \, Al}$$

An analysis of experimental data shows that for the reaction of enrichening the logarithm of the specific activity (Table 4) is a linear function of the aluminium content in the catalysts (quite accurately so for samples Nos. 1, 2, 3, 4), *i. e.* the single ensemble equation is satisfied. In this case, the single ensemble is a group of two aluminium atoms since the predominating ions in the impregnating solutions are those of $Al_2(SO_4)_2$ ([3]), from which, when they decompose on the surface of the silica gel, groups of two aluminium atoms are immediately fixed.



In the figure, besides the experimental points, we have plotted a theoretical, single ensemble curve, for the values of p, calculated from experimental data([1]). It would hardly be of any use to conclude on the basis of the small deviations of catalysts Nos. 5, 6, 7 that there exists a second type of ensemble, seeing that the specificity of catalyst No. 7, the one that shows the greatest deviation, is born out by the fact that the catalyst prepared by impregnating with a 7 N aluminium sulphate solution, and whose aluminium content is almost equal to that of catalyst No. 7, shows a smaller deviation. This also confirms the assumption about the surface of the silica gel being filled with aluminium oxide in a different manner, when highly concentrated solutions of aluminium sulphate are used, than is the case for dilute solutions.

The relation between the logarithm of the specific activity in cracking (Table 4) and the aluminium content in the catalyst is also linear; this confirms the conclusions drawn from the analysis of the study of the activity of catalysts in the enrichening reaction.

Table 4

Summary of Data

No. of catalyst	Normality of impregnating solution	Al content in catalyst	Enrichening			Cracking	
			Iodine number	$a = \frac{\lg \frac{I_0}{I_x}}{\% \text{ Al}}$	$\lg a$	Yield of benzene from gasoil A_{cr}	$\lg a_{cr} = \lg \frac{A_{cr}}{\% \text{ Al}}$
—	0.00	—	78	—	—	0.00	—
1	0.20	0.106	37	3.059	0.485	1.7	1.206
2	0 50	0.339	23	1.563	0.194	4.3	1.104
3	1.75	1.060	18	0.601	$\bar{1}.779$	12.0	1.054
4	4.00	2.644	57	0.051	$\bar{2}.700$	14.0	0.725
5	6.00	3.920	34	0.092	$\bar{2}.963$	13.0	0.521
6	8.00	5.290	28	0.089	$\bar{2}.926$	15.0	0.453
7	12.00	4.390	10	0.218	$\bar{1}.308$	0.0	—

Conclusions. 1. A study has been made of a number of catalysts for the enrichening of benzene and the cracking of gasoil. These catalysts were prepared by impregnating pure silica gels with acidified aqueous aluminium sulphate solutions of various concentrations.

2. For two samples of catalysts, differing widely in activity, it has been shown that the disproportioning of hydrogen proceeds as a reaction of the first order; hence, we have found a method of estimating the activity of a catalyst for the enrichening of benzene.

3. Application of Kobozev's theory for the interpretation of the relation between the activity of the catalyst and its aluminium content shows that the active centres are groups of two aluminium atoms each.

4. Analysis of the activity of these same catalysts in cracking confirms these results.

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REFERENCES

[1] Н. И. Кобозев, ЖФХ, **13**, 1 (1939). [2] G. Jander, K. Jahr, Kolloid Beihefte, **41**, 1 (1934); **42**, 297 (1935); **43**, 295 (1935). [3] H. Denham, J. Chem. Soc. (London), **93**, 41 (1908). [4] N. Bjerrum, Z. f. physik. Chemie, **59**, 350 (1907).

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HYDROGEOLOGY

HYDROGEOLOGY OF CARBONIFEROUS DEPOSITS OF THE VOLGA AREA

By A. B. RONOV

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§ 1. Present Relations. A characteristic feature of the hydrogeology of the southern part of the Volga region is a regular increase in the specific gravity of edge waters with depth, from one aquifer to another. Thus, at Syzran the specific gravity of the waters of the Upper Carboniferous is 1.03; of the Vereyan horizon, 1.045; of the coal-bearing series (on the average), 10.9; of the Upper Devonian, 1.16; and of the Middle Devonian, 1.17.

Quite different is the picture observed in the northern parts of the Volga region, where the mineralization of the waters increases down to a definite level, and after having attained a value close to salt saturation, remains constant with a further increase of depth. As an example may be cited the edge waters of Krasnokamsk, whose specific gravity in the Kungurian horizon is 1.09; in the Upper Carboniferous, 1.18; in the Vereyan horizon, 1.18; in the coal-bearing series, also 1.18.

A study of the mineralization of the edge waters along the strike of the coal-bearing series (C_1^{2k}) and of the Vereyan horizon (C_2^1) makes it possible to distinguish two underground basins within the territory of the Volga region: the northern basin filled with strongly mineralized waters, and the southern basin, characterized by waters of a medium mineralization. West of these is located the western fresh-water basin of the Russian platform. Along the line of the Zhiguli and Pugachev faults passes the area of discharge of highly mineralized waters of the northern underground basin, whose area of head formation seems to be located on the western slope of the Urals (ascending mineral waters of Ussolye and the Big Kushum River). The outlines of the northern basin coincide with the area where the Kungurian gypsum-anhydrite water-resistant horizon has been completely preserved. The specific gravity of the edge waters of the coal-bearing series varies from 1.17 to 1.18.

The Vereyan waters lie nearer the surface and therefore are more susceptible so the facial peculiarities and erosion defects of the Kungurian gypsum-anhydrite teries. This accounts for a certain freshening of the waters in the region of the Sok elevations and at Buldyr, where the Kungurian water-resistant horizon has suffered an Akchagyl erosion. The southern underground basin is located in a zone where this water-resistant horizon has fully been removed by a Mesozoic erosion. The western boundaries of the southern basin pass along the eastern marginal parts of the Voronezh crystalline massif and the Oka-Tzna uplift, whose Carboniferous waters are almost fresh.

A study of the water-resistant horizons located below the Kungurian gypsum and anhydrites shows that in a number of places they have either been removed by erosion, or are pinching out as a result of a facial variability of the rocks, and for this reason they cannot serve as a serious obstacle for a vertical migration of the waters. Thus, *e. g.*, in the Buguruslan region the Vereyan section contains hardly any clays. This is not a unique case. In Tuymaza the thickness of the Vereyan clays does not exceed 2 m, therefore here, too, channel-ways for a vertical migration of the waters might have been formed. At Syzran the clays of the B_2 horizon are occasionally thinning out, producing a natural connexion between the waters of the coal-bearing series and the waters enclosed in the carbonate waters of the Tournaisian and Upper Devonian. It is also known that the Kungurian gypsum and anhydrites are facially replaced eastward of the Volga (the Sok uplifts) by Ufa sandstones.

§ 2. Palaeohydrogeology. The regularities in the present distribution of the edge waters along the vertical and the strike of aquifers are not casual, being related to the geological and hydrogeological history of the Volga region. The nature of the Lower and Middle Carboniferous deposits and fauna indicates that the original salinity of the buried edge waters was not high and corresponded to the salinity of the parent sea basins. A reduction in the size of the Volga subgeosyncline at the end of the Artinskian time (1), in connexion with a growing tendency towards elevation, was accompanied also by the shallowing of the basin. Under conditions of an intense evaporation the sea waters were enriched in salts, and the concentration of the waters increased with time. Highly mineralized sea waters, being more heavy, began to displace the less strongly mineralized edge waters. The process of replacement gradually involved increasingly deeper horizons, until the whole of the Palaeozoic section was filled with a uniformly concentrated Kungurian brine. During their downward movement the sea brines used natural channel-ways in water-resistant horizons connected with their local facial defects. Upon the completion of the process of regional replacement of waters in the drying sea basin, thick layers of hydrochemical sediments were precipitated (P_1^k) which presented a further access of upper waters to the depth. Large-scale processes of cationic exchange started between the waters and the enclosing rocks, as a result of which the waters acquired a chlorine calcium composition, and the rocks suffered an intensive dolomitization.

In the Triassic the Volga region was raised above sea level and became the field of action of erosion processes, which ended in the Middle Jurassic transgression. The removal of the residual cover proceeded non-uniformly and was caused by a non-uniform distribution of the elevation forces. The northern part of the Volga region which was undergoing a slow elevation suffered a slight erosion, while the rapidly rising southern part of the Volga region underwent intensive erosion, as a result of which the Kungurian gypsum-anhydrite horizon was removed. The different course of the tectonic development of the northern and southern regions predetermined the further differences in their hydrogeological regime. In the North of the Volga region, under a protective cover of Kungurian gypsum, highly mineralized edge waters were present in all horizons, while in the southern regions, laid bare by deep erosion, began a gradual dilution of edge waters by comparatively slightly mineralized waters of Mesozoic and Cenozoic seas. The percolating waters were less heavy and therefore less gravitationally active. As a result, the process of replacement was very slow and was becoming less intensive with depth. This was the cause of the regular increase in the specific gravity of edge waters observed at present from the higher to the lower horizons.

§ 3. Processes of Formation of Gas Fields. As distinguished from processes of coal and oil formation, which are related to a metamorphism of vegetable remains in terrigenous deposits, the processes of gas accumulation are more universal. On the one hand, the formation of gases accompanies the formation of coal and oil; on the other hand, its proceeds also in thick deposits of carbonate rocks. Each of the innumerable shells becomes the field of

transformation of organic matter to gaseous hydrocarbons. Leaving the parent substance, the gases penetrate the edge water, become dissolved in it, and their subsequent fate depends on the change in the pressure of the strata and the concentration of salts in the waters, which control the solubility of the gas.

At the same time with the variation in the composition of the waters, closely depending on it, as well as on the oscillatory movements of the Earth's crust and the growth of domes, proceeded the formation and further evolution of the gas fields of the Volga region. During the periods of a general elevation of the country, the pressure of the edge waters decreased, and the gases from a dissolved state passed to free accumulations under gas-proof horizons. The increase of mineralization of the waters acted in the same direction. During the periods of a regional subsidence of the Earth's crust the pressure of edge waters increased, and the gases returned to the dissolved state. A similar effect was produced when the mineralization of the waters decreased, for their ability to dissolve gases increased at the same time.

During the formation of the coal-bearing series, at the period of a general elevation of the Volga region, the gases dissolved in Devonian waters were evolved from the solution and collected in the most raised areas—under Devonian gas-proof clays. The general subsidence which followed (Visean age) brought about an increase of the pressure of edge waters, and the gases from a free condition returned to the dissolved state. The same phenomenon occurred during the Visean period of elevation of the country and the subsequent Middle and Upper Carboniferous subsidences. However, this time the evolution of gases from solution and their subsequent return to the dissolved state affected at once several stages of the section: the Devonian, Visean and coal-bearing series. Each time when the gases formed free accumulations, a certain part of the gas molecules, escaping through local defects in the gas-proof horizons, migrated upwards and accumulated under the next gas-proof horizon. The processes of evolution of gases from the solution became gradually more intensive beginning with the Kungurian age, in connexion with their penetration to the depth of highly mineralized waters, as well as with the fact that in a number of places the general gradually increasing regional elevation of the Volga region was superimposed by local elevations, connected with the growth of domes ([2],[3]). By the beginning of the Mesozoic under the Kungurian gas-proof rocks large masses of gas were accumulated. In the northern regions where the Mesozoic erosion did not affect the integrity of the Kungurian, the hydrocarbon gases remained in place (Buguruslan, Tuymaza, Krasnokamsk, *etc.*), having only later suffered the destructive action of desulphurization processes (Buldyr, Baitugan). In the southern regions the accumulations of gases under Kungurian gypsum escaped into the atmosphere simultaneously with the decomposition of the gypsum, and were preserved only under Vereyan and coal-bearing clays (Elshanka, Vorontzovka, Teplovka).

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REFERENCES

[1] V. V. Beloussov, Bull. Acad. Sci. URSS, sér. géogr. et géophys., No. 4,533 (1940). [2] A. B. Ronov, C. R. Acad. Sci. URSS, XLIV, No. 4 (1944). [3] A. B. Ronov, ibid., XLIII, No. 1 (1944).

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MINERALOGY

BLUE HALITE FROM THE SOLIKAMSK POTASSIUM DEPOSITS

By V. A. APRODOV

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The sylvinite zone of the Solikamsk salt deposit is characterized by an extreme variety of the coloration of the component salts. This is especially true of the colours of halite which occurs in the form of ice-transparent, white, smoky, yellow, pink, dark-blue, light-blue or greenish varieties. The light-blue and dark blue halites stand out with particular clearness; it is to them that sylvinite owes the «marble» pattern which distinguishes the Solikamsk specimens.

During his visit to the first potassium mine at Solikamsk the author became aware of a number of peculiarities in the distribution of dark-blue halite. The latter is associated in the rock with sylvite and marl; and less frequently, with carnallite. In small crystals enclosed in milk-white sylvite, the blue colour of the halite is concentrated not far from the contact of the minerals. It vanishes rapidly towards the centre of the small cubes of halite, and forms a distinctly delimited zone up to 0.2 mm thick. Between this and the sylvite there is always a thin zone of colourless halite. Chirvinsky ([8]) identified these coloured zones with skeletons of crystal growth.

In large colourless crystals of halite the blue coloration often appears near deformed crystals of milk-white sylvite. The sylvite here shows traces of a lit-par-lit flowage with a plastic deformation of definite portions of it intruded in halite. At a certain distance from them inside the halite the blue coloration is noted in the form of oval and crescent spots with diffuse edges. There are also cloud-shaped and flaky areas of light-blue halite, deformed together with the colourless mineral. In the intensely blue halite dark «centres» are observed, occasionally brown; nearly black in colour. Areas of this kind are sharply delimited from the colourless halite. In a few rare cases the direction of cleavage within them makes an angle of 45° with the cleavage of the colourless crystals. At the same angle are also the boundaries of the dark-blue areas with regard to the outlines of large halite crystals. It seems that the dark-blue halite represents separating portions of crystals turned with respect to the enclosing ones. Such a reorientation has repeatedly been noted for crystals of halite, subject to a strong pressure. Thus, all these phenomena go well to supplement one another. However, the distribution of the dark-blue colour in the crystals is an evidence of the fact that in a number of cases deformation took place after they had assumed their colours.

Passing to larger volumes of rock, it may be noted that the dark-blue colour of the halite appears in the vicinity of bands of red sylvite and, less frequently, near inclusions of marl. This is also evidenced by data of core drilling. In some cases the coloration of the light-blue band is due to an abundant dissemination of small (up to 1 mm in size) crystals of a dark-blue and black halite. The follow-

ing alternation of bands is commonly observed: a red band of sylvite, a dark-blue, finely disseminated, and then a bluish-grey, smoky one. The bands are overlapping, owing to which their boundaries are somewhat diffuse. Sometimes grey halite becomes smoky or yellowish with a faint violet shade, resembling fluorescence. In such a halite the angles of the crystals turned to the sylvite band are faintly blue in colour. The boundary of the blue areas in each crystal makes with its faces an angle of 45°.

The banding of such a type may reoccur. In cores of small folds of salt the thickness of the blue bands increases at a much higher rate than that of the grey and white ones. This seems to be due to a localization of pressure here, and possibly, in part to a lesser limiting load necessary for the blue halite to pass to the liquid state.

Mention should also be made of a well expressed concentration of blue halite in places of particularly strong dislocations, accompanied by fractures in the salts enclosing the sylvite.

It is commonly believed that the blue colour of halite appears when its sodium ions are neutralized ([5],[6]). Artificial blue coloration of halite was produced by exposing it to the action of cathode rays or of radium β-rays ([6]). It is of interest to note that the originally appearing yellow hues of the salt exposed to the rays gradually pass through a series of shades to violet and blue ([6]). The smoky yellowish and blue varieties of the Solikamsk halite can therefore be linked into a single genetic series of phenomena. From this standpoint noteworthy is the green colour of the halite, observed in drill-hole No. 8 of the Solikamsk prospecting party, at a depth of 169.6 m at the contact with flesh-red carnallite ([3]). It may be considered to be the extreme step in the decrease in the intensity of the original yellow colour ([6]). On the other hand, in the most widespread radioactive processes, α-conversion accompanies the coloration of the minerals in brown and black hues, the appearance of smoky colour ([10]). The appearance in the minerals of pleochroic petrographic haloes may be due to the presence of radioactive inclusions. From this standpoint there is a certain analogy between the brown and black spots in the blue halite and petrographic haloes.

The obvious genetic relation of the blue colour of halite to the brown and black spots within it is striking. All these changes in the colour of the salt are quite similar to its coloration upon being exposed to two different radioactive radiations.

The intimate relation between blue halite and sylvite suggests the idea that one of the causes of the appearance of the blue colour may be the potassium present there. The chemical composition of the Solikamsk blue halite as given by Chirvinsky ([8]) is 0.54 per cent of potassium and rubidium chlorides. Potassium and rubidium stand close to each other in the periodic table; they are radioactive minerals, readily emitting electrons, and with a radioactive transformation of the β type ([5], [7]).

Of great interest in this connexion are the observations made by Fersman ([6]) concerning the transformation of transparent quartz to smoky quartz near its contacts with orthoclase, as a result of a radioactive effect of potassium on quartz. The presence of a radioactive admixture in the brown and black portions of halite easily explains its blue colour as a secondary phenomenon. It should also be mentioned that the red sylvite is intimately associated with an inclusion of marl in the rock. Its red colour is produced here by the presence of finely disperse haematite ([4]). Marl is, possibly, one of the factors causing intense coloration in salts. It is worthy of mentioning that according to data by Razumovskaya([4]) a constant increase in the helium and neon content is observed in gases enclosed in the Solikamsk salts. A similar fact enabled Erdman (in 1910) to arrive at the conclusion that the formation of haematite, colouring the potassium salts of the Stassfurt deposit, proceeded under participation of radioactive elements. On the other hand, the intensity of the blue colour of halite and the number of its coloured crystals rapidly increase in strongly dislocated portions of the deposit.

The same phenomenon was noted by the present author in the microdistribution of blue halite in the rock. On the cleavage planes of large crystals of colourless halite in which its blue varieties are enclosed, under the magnifying lens «etching patterns» of a fluidal appearance are visible. They have either a fan-shaped, or a dendritic arrangement. The orientation of these ripple-like etchings on the crystal faces corresponds to the direction of pressure in the deformed portion of the rock, and is due to the migration of the gaseous substance from deeper zones along fissures. It should be noted that at such places the colourless halite crystals are particularly large, occasionally attaining 10—15 cm across. This increase in size may also be explained by radioactive influence which is known to enhance the crystallization capacity of salts. The secondary nature of the radioactivity of potassium in blue halite and in sylvite may be presumed also on account of the effect of radioactive substances migrating along the fissure from deeper zones. An analogy to this phenomenon is presented by the radioactivity of waters in the springs of Tartaria, which is related to the fracture tectonics of platform structures ([9]).

The peculiarities in the tectonics of platform oil-bearing structures of the near-Kama area made the author refer them to a separate type characterized by an upward migration of matter, a peculiar transformation of the vertical movements of the lithosphere ([1], [2]). Recording the existence of radioactive phenomena in the oil waters of the Pechora and the near-Kama area, it seems not amiss here to point out the relation of the blue colour of the halite of the Solikamsk potassium deposit. From its tectonic structure the region of this deposit shows a marked likeness to certain large structures of the Molotov region in the near-Kama area, being, however, broken by relatively great disjunctive dislocations. The existence of elements of evident diapir and salt-dome tectonics adds much interest to the problems of the analysis of the tectonics of deep zones in this part of the cis-Urals downwarping, as one of the presumably petroliferous territories.

It is hoped that a detailed study of the peculiarities in the distribution of blue halite in the Solikamsk deposit will make possible the solution of many important problems concerning the structure of this region, which is of great interest both from the geological and the geochemical standpoint. In particular, it would contribute to the discovery of the source of accumulation here of a huge mass of salts and, possibly, furnish guiding data for the organization of deep drilling.

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REFERENCES

[1] V. A. Aprodov, C. R. Acad. Sci. URSS, XL, No. 5 (1943). [2] V. A. Aprodov, Bull. Acad. Sci. URSS, sér. géol., No. 2 (1943). [3] Геологический комитет, Материалы по исследованию Прикамского соленосного района, вып. VII; Материалы по прикладной геологии, вып. 125, 1929. [4] Е. И. Разумовская, Материалы по исследов. Прикамск. соленосного района, II, 1927. [5] Спутник химика, 4, вып. 1, 1940. [6] А. Е. Ферсман, Цвета минералов, изд. АН СССР, 1936. [7] А. Е. Ферсман, Геохимия, 4. 1939. [8] П. И. Чирвинский, Зап. Всеросс. мин. об-ва, II, сер. 72, в. 1 (1943). [9] V. Cherdyntzev, C. R. Acad. Sci. URSS, XXXVI, No. 7 (1942). [10] C. Doelter, Das Radium und die Farben, 1910.

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MINERALOGY

BIREFRINGENCE OF ANY RANDOM SECTION OF AN OPTIC INDICATRIX

By L. A. VARDANJANZ

(Communicated by D. S. Beliankin, Member of the Academy, 27. IX. 1944)

In the course of theoretical investigations it may occasionally be found necessary to calculate the birefringence of any random section of an optical indicatrix. In the general case, two equations must be solved jointly, namely, the equation of the indicatrix $\frac{x^2}{N_p^2}+\frac{y^2}{N_m^2}+\frac{z^2}{N_g^2}=1$ and the equation of the plane of a given section $ux+vy+wz=0$, where u, v, w are the cosines of the angles between the normal to the given plane and the coordinate axes. The equation $u^2+v^2+w^2=1$ should always hold true. In Rosenbusch' textbook the problem is solved in the form of the following equation

$$\lambda^2(u^2N_p^2+v^2N_m^2+w^2N_g^2)-\lambda[u^2N_p^2(N_m^2+N_g^2)+ +v^2N_m^2(N_p^2+N_g^2)+w^2N_g^2(N_p^2+N_m)]+N_p^2N_m^2N_g^2=0 \quad (1)$$

which in its concise form may be written as a simple square equation

$$a\lambda^2-b\lambda+c=0 \quad (2)$$

The roots of this equation, which are the vectors of the indicatrix, equal:

$$\lambda_1=\frac{b+\sqrt{b^2-4ac}}{2a};\quad \lambda_2=\frac{b-\sqrt{b^2-4ac}}{2a};\quad N_g'=\sqrt{\lambda_1};\quad N_p'=\sqrt{\lambda_2} \quad (3)$$

From equations (3) the following equation may be constructed

$$N_g'-N_p'=\sqrt{(\sqrt{\lambda_1}-\sqrt{\lambda_2})^2},\ \text{or}\ N_g'-N_p'=\sqrt{\frac{b-\sqrt{4ac}}{a}} \quad (4)$$

In spite of the very simple form of equation (4), its solution presents considerable difficulties. First of all, let us determine the approximate value of the coefficients a, b, c in equation (4). For refringence of from 1.5 to 2, coefficient a equals from 2.25 to 4; coefficient b, from 10 to 32; and coefficient c, from 11.5 to 64, while $\sqrt{4ac}$ will approximate $10-32$. With $N_g-N_p'=0.001$, $b-\sqrt{4ac}$ should equal $0.000002-0.000004$. In order to find this difference with sufficient accuracy, the values of $\sqrt{4ac}$ and b must be calculated with an error not exceeding ± 0.000002, and this is far from being practicable, even with the aid of seven-place logarithms. When $N_g'-N_p'\cong 0.0001$, the problem can be solved only with nine-place logarithms. If five-place logarithms are used, this problem can be solved only for $N_g'-N_p'\cong 0.01$.

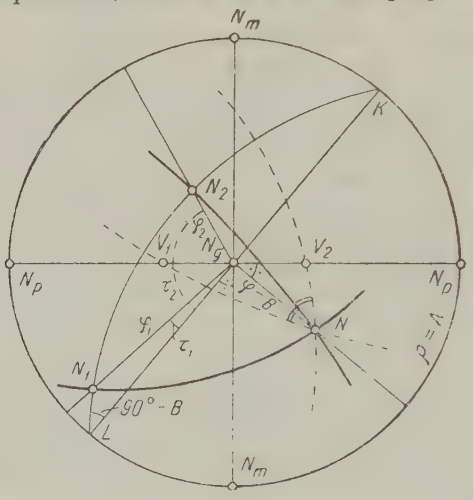
It might be attempted to solve equation (4) by the usual means of arithmetical calculation, but this method also fails to give the desirable effect. The trouble is that the coefficients of light refringence N_p, N_m, N_g are

numbers with four or even five decimal places, and with five or six significant digits, while the values of the cosines u, v, w have no less than four decimal places, and at least one of the cosines must have less than six decimal places. The error of equation $u^2+v^2+w^2=1$ makes it impossible to solve the fundamental equation (1). In order to determine under such conditions the precise value of coefficient b, it would be necessary to calculate a number having at least 20 or 22 significant digits, while for the product $4ac$ this number would have to be raised to 40 or 44.

The author has evolved another, simpler equation, which is derived in the following manner. The equation of the indicatrix is expressed in polar coordinates

$$\frac{N^2 \sin^2 \varphi \cos^2 \rho}{N_p^2} + \frac{N^2 \sin^2 \varphi \sin^2 \rho}{N_m^2} + \frac{N^2 \cos^2 \varphi}{N_g^2} = 1 \qquad (5)$$

where the equatorial plane is $N_m N_p$ (see figure). LN_1N_2K is the plane of the indicatrix section in question, and N is a vector perpendicular to the latter,



with the polar coordinates: longitude $\rho = A$ and polar distance $\varphi = B$. In order to solve our problem, the vectors $N_1 = N_p'$ and $N_2 = N_g'$ must be found in the plane LN_1N_2K. For vector N_1, with the coordinates $\varphi = \varphi_1$ and $\rho = 90° + A + \tau_1$, equation (5) becomes

$$N_1 = \frac{1}{\sin^2 \varphi_1 \left[\frac{\sin^2 (A+\tau_1)}{N_p^2} + \frac{\cos^2 (A+\tau_1)}{N_m^2} + \frac{\operatorname{ctg}^2 \varphi_1}{N_g^2} \right]} \qquad (6)$$

From the spherical triangle LN_1N_g: $\operatorname{tg} \varphi_1 = \frac{\operatorname{ctg} B}{\sin \tau_1}$, or $\operatorname{ctg} \varphi_1 = \sin \tau_1 \operatorname{tg} B$. Hence

$$\sin^2 \varphi_1 = \frac{1}{1 + \sin^2 \tau_1 \operatorname{tg}^2 B}$$

Substituting these values in equation (6), we get

$$N_1^2 = \frac{1 + \sin^2 \tau_1 \operatorname{tg}^2 B}{\frac{\sin^2 (A+\tau_1)}{N_p^2} + \frac{\cos^2 (A+\tau_1)}{N_m^2} + \frac{\sin^2 \tau_1 \operatorname{tg}^2 B}{N_g^2}} \qquad (7)$$

Similarly (see figure)

$$N_2^2 = \frac{1 + \sin^2 \tau_2 \operatorname{tg}^2 B}{\frac{\sin^2 (A+\tau_2)}{N_p^2} + \frac{\cos^2 (A+\tau_2)}{N_m^2} + \frac{\sin^2 \tau_2 \operatorname{tg}^2 B}{N_g^2}} \qquad (8)$$

In both cases, the value of vector sought for is a function of a single variable τ_1 or τ_2, which can be determined as a function of the arc N_1L, namely:

$$\operatorname{tg}\tau_1 = \operatorname{tg} N_1L\cos B \quad \text{and} \quad \operatorname{tg}\tau_2 = \operatorname{tg} N_2L\cos B = -\operatorname{ctg} N_1L\cos B \tag{9}$$

In its turn, the value of the arc N_1L (or of the angle N_gNN_2 equivalent to it) can be determined from spherical triangles V_1N_gN and V_2N_gN by means of the equations:

$$\operatorname{tg} N_1L = \operatorname{tg}\frac{V_2NN_g - V_1NN_g}{2} \tag{10}$$

$$\operatorname{tg}(V_2NN_g - V_1NN_g) = \frac{\sin 2A\cos 2B}{\operatorname{ctg}^2 v\sin^2 B - \cos^2 B\cos^2 A + \sin^2 A}\,. \tag{11}$$

Solving (10) and (11) with the aid of five-place logarithms, we get a value for arc N_1L with its error not exceeding a few seconds, and then the values for τ_1 and τ_2 with the same accuracy.

Equations (7) and (8) contain three terms each with trigonometric functions. Two of these, $\sin(A+\tau)$ and $\cos(A+\tau)$, are always less than unity. The third term, $\sin\tau\operatorname{tg} B$, is also always below unity, since $\sin\tau\operatorname{tg} B = \operatorname{ctg}\varphi$ and $\cos\varphi = \sin B\sin NL$; hence, with $B < 45^\circ$ we have $\operatorname{tg} B < 1$ and $\operatorname{ctg}\varphi < 1$, and with $B > 45^\circ$ we have $\cos\varphi < \sin NL$, $NL < 45^\circ$ and $\operatorname{ctg}\varphi < 1$. As to the coefficients N_p, N_m, N_g, they represent numbers from 1.4 to 2, with four or five decimal places, *i. e.* with only six significant digits.

Hence, using five-place logarithms to solve equations (7) and (8), we can find the value of each of their terms in the numerator and the denominator with an error in the seventh decimal place only; thus, we are able to find the values for N_1 and N_2 with an error not exceeding 0.00001, even in the most unfavourable case of accumulation of errors in the process of taking logarithm. The birefringence can be found as the difference between N_1 and N_2, with about the same accuracy. Therefore, it may be concluded that the limit value of birefringence $N'_g - N'_p$, which can be calculated using the author's equation even with the aid of five-place logarithms, and with a quite satisfactory accuracy at that, is equal to 0.0001—0.0002.

To check this conclusion, calculations were made with the aid of five-place logarithms for a positive mineral with $2V = 5^\circ$, $N_g = 1.6474$, $N_p = 1.6374$ and $N_m = 1.637419$, and for a positive uniaxial mineral very closely approaching it, with $N_g = 1.6474$ and $N_p = 1.6374$, with $A = 40^\circ$ and $B = 10^\circ$. For the uniaxial mineral it was found that $N'_g - N'_p = 0.00031$, and for the biaxial, $N'_g - N'_p = 0.000305$.

Thus, the correctness of the above conclusion as to the accuracy of the solution of the problem in question with the aid of this equation is fully established, since under definite (and at the same time arbitrary) conditions the birefringence of a uniaxial and biaxial mineral should be almost identical, being somewhat lower in the biaxial than in the uniaxial mineral.

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PALAEONTOLOGY

ON BRYOZOAN AND CRINOIDAL REEFS OF THE PERMIAN PERIOD IN THE URALS

By N. N. YAKOVLEV, Corresponding Member of the Academy

In 1944 the author stayed some months in the town of Krasnoufimsk in the Urals, as consultant to the expedition from the Oil Institute, Leningrad; he thus had the opportunity to study in detail the structure and fauna of the local reefs connected with oil fields.

Structure and Composition of Reefs. The Lower Permian reefs, widely distributed in the Urals, are represented in Krasnoufimsk by hills or chains approaching by their form the shape they had at the time of their formation at the sea bottom. They include a limestone core, with a comparatively thin cover around it (several metres thick), consisting of greenish-grey marls.

It should be erroneous to confuse the fauna of the two parts of the reef, as it is quite different: in the core its character most frequently depends on bryozoans, whereas for the marls it is determined by abrachiate and few-brachiate crinoids.

The fauna of sea lilies is nearly absent in the limestone core of the reefs, and bryozoans play a most insignificant rôle in the marly cover of the core.

The marls include an abundant fauna of crinoids; especially characteristic are: *Hemistreptacron abrachiatus* Yakov., *Proindocrinus piszowi* Yakov., *Camptocrinus permiensis* Yakov., *Calycocrinus rossicus* Yakov., *Kallimorphocrinus uralensis* Yakov., and the blastoid *Sagittoblastus wanneri* Yakov. These echinoderms are present in all the recorded outcrops of marls in Krasnoufimsk and its vicinities, and besides, were also found by N. Dorofeev during the war time north of Cherdyn on the Kolva River (Vetlan and Boets hills). Besides sea lilies, characteristic are the following (not abundant) corals which are easily determined: *Cladochonus*, *Pseudofavosites*, *Oculinella*; rare and small brachiopods *Hustedia remota* Eichw.; there are also present siliceous sponges *Pemmatites* and *Anthracosycon* (?); gastropods and conchifers are rare and represented by a few species.

As to the limestone core of the reef, as for instance the reef of Mount Divia in Krasnoufimsk, it is on the base of its fauna subdivided into two horizons—the lower, containing from bryozoans only *Cryptostomata* (*Fenestellidae*), and the upper, in which *Trepostomata* (*Batostomellidae*) are prevailing.

Besides, the limestone core of Mount Divia contains the following brachiopods (according to the collections and determination of the author): *Productus scabriculus* Mart., *Pr. fasciatus* Kut., *Poikilosacos tschernyschewi* Fred. (established for the first time on the base of its find in this locality), *Pr. stuckenbergianus* Krot.,

Pr. janischewskianus Step., *Spiriferina ornata* Waag., *Martinia uralica* var. *longa* Tschern. (the author does not mention the rhynchonellids which are of less importance), *Amplexus* sp. (overfills the layer in the upper part of the mountain), *Aviculopecten subclathratus* Keys.

In the lower part of the limestone deposits was found the trilobite *Cheiropyge maureri* Weber.

In their uppermost part the limestones of Mount Divia are poor in organic remains (mentioning deserve *Favosites*).

An elucidation of this section of Mount Divia was mainly possible owing to a ditch digged out for a water conduit.

The reef of the Sarga River, located on the opposite side of the railroad tunnel Mount Divia, includes the following fauna: *Poikilosacos*, *Productus transversalis* Tschern., *Pr. gruenwaldti* Krot., *Pr. scabriculus* Mart., a new species of *Choristites*, approaching *Ch. nikitini* Tschern., but differing from the latter in its still shorter hinge margin, its length in relation to the breadth of the shell being equal to 0.4, whereas in *Ch. nikitini* this proportion is equal to 0.5. The author proposes for this form the name of *Ch. postnikitini*. In the limestone of the Sarga River there is also present a large form of *Dielasma*, *Amplexocarinia*, *Aviculopecten subclathratus* Keys.

The limestones are characterized by a rapid cementation of the rock made evident by the fact that the interior part of the brachiopod shell and of polyparies of *Rugosa* corals is not filled by rock.

Biological Peculiarities Characteristic of the Reef Fauna of Crinoids. With reefs, rising rather steeply from the sea bottom, was undoubtedly connected the existence of the surf. The surf had a marked influence on the character of the fauna in the marlaceous parts of the reef.

In marls there is observed a prevalence of mainly abrachiate and few-brachiate forms of the crinoid fauna, namely, abrachiate forms: *Hemistreptacron*, *Streptostomocrinus*; forms with 3 arms: *Proindocrinus*, *Sundacrinus*, *Halysiocrinus*; with one arm: *Monobrachiocrinus*. The reduction of arms and the development of the calico in some sea lilies (*Calycocrinus*, *Halysiocrinus*), more on the one side than on the opposite, can be explained by their existence in the zone of the surf, where abundant food is brought by the current (microplankton); here, the arms may be reducedas food-getting devices with economy and benefit for the entire organism. The oblique position of the peristomal field of the calice (due to its unequal sides) and sometimes a constant inclination of the stem in the same direction, due to unequal sides of its columnals, are favourable for the catching of prey. The stem sometimes consists (*Calycocrinus*) of elongate and probably not numerous columnals and is reduced in length, which makes it more resistant to disruption during the action of the surf.

In this respect an extreme degree of development is attained by *Brachypus* deprived of stem and adhering to the substratum by the base of the calice.

When the calice was oblique and thus the conditions contributing to the development of arms of an individual were different, the reduction of these arms took place by means of decrease of their number, or there was observed an equal decrease in size of all arms which sometimes were totally atrophied. Taking into consideration the genetic series: *Hemiindocrinus* Yakovl., *Proindocrinus* Yakovl., *Indocrinus* Wanner, established by the author, he comes to the conclusion that the decrease in the number of arms and the relaxation of the respiratory function connected with it was compensated for in this series by a development of thin-walled depressions in the angles of plates of the calicos in *Proindocrinus* and *Indocrinus*; owing to these depressions, the diffusion of oxygen could take place. The development of few-brachiate and abrachiate sea lilies and in general the small sizes of representatives belonging to the fauna of marls in comparison with fauna of deeper seated limestones, can be accounted for by smaller contents of lime in sea water during the deposition of marls. The fauna of sea lilies disappeared beginning with the time

when reefs had ceased to exist in the Urals as a prominent element in the relief of the sea bottom, with the surf around them.

This fauna migrated to the Timor Island where somewhat later it thrived during te Permian time.

Biologically, the fauna of the marls under consideration is similar to that of the Upper Jurassic marls in Moravia (Stramberg), where sea lilies (*Eugeniacrinidae*) have similar habit of stem, with a constant curvature towards one side and an unequally sided calice. Both the Permian and Jurassic periods have a peculiar composition of their fauna. Among animals attached to the place of their existence prevail sea lilies, corals (although in the Urals the latter are comparatively few in number in limestones) and sponges; among slowly progressing animals, echinoids (*Cidaridea*).

The fauna of Stramberg differs from that of Krasnoufimsk by the absence of few-brachiate and abrachiate crinoids. Such crinoids are also known from the Upper Palaeozoic of North America.

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REFERENCES

[1] O. Jaekel, Z. Deutsch. Geol. Gesellsch., XLIII, H. 3 (1891). [2] Н. Яковлев, Изв. Геолог. ком., XLV, No. 2 (1926); XLVI, No. 3 (1927); Изв. ГГРУ, XLIX, No. 8 (1930); Ежегодник Всерос. палеонтол. общ., XI (1938).

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GENÈTICS

SUSCEPTIBILITY OF INTESTINE EPITHELIUM IN MALES AND FEMALES OF *DROSOPHILA MELANOGASTER* TO THE NOXIOUS EFFECT OF LACTIC ACID

By P. G. SVETLOV

(Communicated by A. A. Zavarzin, Member of the Academy, 22. X. 1944)

When studying the ontogenesis of the sex-differential of susceptibility in *Drosophila melanogaster*, even in young larvae the cells of imaginal discs in males or females have been found to show a different degree of susceptibility to various noxious agents, those of the males being much more susceptible ([3]). The same has also been found with respect to the imaginal discs of young chrysalides (unpublished data). These data do not settle, however, the question as to whether and to what extent the difference in susceptibility observed between males and females of the imaginal instar and in chrysalides ([1], [2]) may be accounted for by their tissue susceptibility alone. We can imagine *a priori* that the higher susceptibility in males is due to anatomical differences or physiological peculiarities of correlative nature, rather than directly to a higher susceptibility of their tissue elements, in other words, that these differences are of «epidesmal» character.

In order to shed more light upon this question, experiments were organized, involving determination of susceptibility of the very tissue elements in *D. melanogaster* at the imaginal instar. Like in our work with imaginal discs, the method of observation in the dark field was applied. The intestine of the flies was chosen for the investigation as the most convenient material. Flies 1—2 days upon hatching were dissected in a saline solution (drosophilic Ringer-solution), and the anterior part of the alimentary channel, with the gut and proventriculus were extracted under maximum precautions. For examination invariably served a small portion of the intestine, directly adjoining the proventriculus from behind; this is a thin cylindrical tube, consisting of flat monolayer epithelium. The contents of the intestinal tubule comprise no solid particles in this section, being therefore optically void. This section of the intestine is also most convenient because of its total lack of tracheae. Somewhat posteriorly, it is already covered with a dense network of tracheae, and observation in the dark field is hindered by an intense glitter of the tracheal threads.

For the examination served an object-lens $\times 20$ and an eye-piece $\times 10$, with a Zeiss paraboloid-condensor. Using this magnification, cells whose boundaries are outlined by luminescent lines, while the contents are transparent, are clearly visible in the wall of this section of the intestine. But the picture cannot be reproduced unless the intestine is quite intact. An insignificant distention made when dissecting this part of the intestine results in the appearance of a large number of injured (glittering) cells. To eliminate excessive pressure caused by the covering glass, and

to equalize the experimental conditions, I used preliminarily to place on the object-glass two segments of human hair washed with ether.

After being satisfied of the fitness of the material, a drop of 0.0047 per cent lactic acid (14 per cent acid diluted 3000 times) was sorbed under the covering glass with the aid of a strip of filtering paper. At the same time a stop-watch was set operating. The supply of lactic acid was continued throughout the experiment. Lactic acid was made use of as a reagent which permits to observe a highly distinct and fine picture of plasmic injuries. Soon after the experiment was started (within the first minute, as a rule), opalescence of the whole plasmic background sets in. Further on opalescent nuclei appear, and the wall of the intestine assumes the aspect of a fixed preparation. Subsequently the plasm begins to emit a still more intense luminescence, owing to which the nuclei become scarcely observable again. Along with this, the whole body of the cell starts shrinking, so that dark fissures appear between the glittering cells. Finally, all the cells of the intestinal epithelium are found to be isolated from one another, while their contents become homogeneous, and emit an even luminescence. Among the stages of this process, it is the appearance of isolated cells over a great part of the preparation that can most conveniently be recorded at its very onset. This moment is very constant, it can easily be determined, and individual errors due to subjective perception cannot exceed 5—10 seconds. The data obtained were treated with an accuracy up to 0.25 minutes. The temperature of the room was taken into account; it fluctuated within a range from 18 to 20° C. Experiments were carried on daily, and precautions were taken that the material be composed of males and females at about the same ratio. Males and females were taken of as similar size as possible, *i. e.* big males and small females were selected, always from the same culture, of about the same age and density of population.

A total of 18 experiments with males and as many with females were carried out. The results are summarized in the table.

Intestine of *Drosophila melanogaster* as Affected by Lactic Acid (0.0047%)

Date of exper.	No. of exper.	Time of appearance of isolated cells, in min.	Date of exper.	No. of exper.	Time of appearance of isolated cells, in min.
16.V	1	4.75	16.V	1	12.00
18.V	2	5.50	17.V	2	9.00
	3	4.25	18.V	3	5.00
19.V	4	3.25		4	10.00
	5	4.75	19.V	5	4.00
20.V	6	3.00		6	7 00
22.V	7	3.25	20.V	7	7.00
24.V	8	8.25	24.V	8	9.25
	9	6 50		9	5.75
	10	5.75		10	8.25
29.V	11	6 25	29.V	11	9.25
	12	6.50		12	10.00
	13	6.25		13	4.25
	14	7.50	30.V	14	12.25
30.V	15	6.00	31.V	15	7.00
31.V	16	3.00		16	7.75
1.VI	17	6.50	1.VI	17	10 50
	18	7.75		18	11.75
	Mean	5.47±0.38		Mean	8.11±0.56

$M_{♀} - M_{♂} = 2.64 \pm 0.67$ minutes

As may be seen from the table, the cells of intestine epithelium are much sooner injured in males than in females. The transgression observed in the two columns of figures is, however, a rather important one; it can, therefore, by no means be asserted that the intestinal cells in any male are more susceptible to the effect of lactic acid than those of any female (in males the extreme variants are 3.00—8.25 minutes, while the respective figures for females are 4.00—12.25). The same is true of the susceptibility of the flies to starvation and other noxious agents ([1]). The mean values, however, even when derived from a rather small number of experiments, leave no room for doubt as to the existence of a significant difference in the susceptibility of males and females. The difference between the means for males and females exceeds the difference error almost four times. It should be pointed out in this place that, like in our preceding experiments, the females displayed again a much wider range of variation than the males: $\sigma_{♂} = 1.63$, $\sigma_{♀} = 2.39$. This phenomenon stands in need for further analysis.

Thus, the cells of the first organ of *D. melanogaster* taken at random have been found to be more susceptible to the effect of the noxious agent applied in males than in females. It would be of interest, of course, to test in the same way some other organs, too. It will perhaps prove practicable, although considerable technical difficulties have to be surmounted. Some other animal species should likewise be investigated at this angle. No far-reaching significant conclusions can otherwise be ventured. Yet, even at the time being, it can be stated with assurance that the difference in the susceptibility of the two sexes in *D. melanogaster* to starvation and other noxious agents is based upon differences in their tissue susceptibility. Thus, a step forward has been made in the knowledge of the nature of this phenomenon.

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REFERENCES

[1] P. G. Svetlov, C. R. Acad. Sci. URSS, XLI, No. 8 (1943). [2] P. G. Svetlov, ibid., XLI, No. 9 (1943). [3] P. G. Svetlov and O. V. Chekanovskaya, ibid. XLVI, No. 7 (1945).

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PLANT PHYSIOLOGY

ON THE SOLUBILITY OF BORON COMPOUNDS IN THE PLANT

By E. V. BOBKO and T. D. PRIADILSHCHIKOVA

(Communicated by D. N. Prjanischnikow, Member of the Academy, 1. XII. 1944)

It has been demonstrated in studies carried out at this Laboratory at a previous date that a considerable fraction of boron is contained in the plant in the form of sparingly soluble compounds which do not pass into solution even after six extractions with hot water with the aid of an extractor of periodic action. These former results refer to clover and flax. The work was continued in the current year on the narrow-leaved blue lupine and buckwheat. Lupine was analysed at three dates, *viz.* at the beginning and at the end of blooming, and at maturity. Instead to repeat extraction six times, as in our previous studies, the number of extractions was limited to four. By reducing the number of extractions and thus increasing the content of boron in the residue we hoped to bear out more accurately some differences in the solubility of boron, if any there was, as connected with the part of the plant taken for analysis, or the age of the plant under test. Experience has shown, however, that after four extractions the main part of the boron contained in the plant was found to have passed into solution. The results of determinations are presented in Table 1.

Table 1

Solubility of Boron-bearing Compounds in the Plant

Plant species and parts analysed	Extract		Pulp		Total
	mg/kg	per cent	mg/kg	per cent	mg/kg
Lupine at budding:					
stems	21.4±0.6	82.4	4.6±1.7	17.3	26.0
leaves	28.0±0.1	86.5	4.4±0.4	13.5	32.4
flower-buds	47.8*	86.5	7.5*	13.5	55.3
Buckwheat at the beginning of blooming:					
stems	28.1±1.2	82.0	6.2±0.4	18.0	34.3
leaves	34.3±2.0	84.1	6.5±0.2	15.9	40.8
flowers	26.4±2.3	77.9	7.5±0.4	22.1	33.9
Buckwheat at the end of blooming:					
stems	26.6±1.9	81.1	6.2±0	18.9	32.8
flowers	41.2*	92.8	3.2±0.1	7.2	44.4
Buckwheat at maturity:					
leaves	37.8±0.1	80.3	9.3±0.3	9 7	47.1

* A single determination made.

Before the analysis the material to be studied was divided into stems, leaves and flowers, and these portions were analysed separately, boron being determined in the extract and in the pulp. Unfortunately, some of the paired analyses proved to be contradictory, and could not be repeated for lack of material. Some of the sets are therefore lacking data for all three parts of the plant.

As may be seen from Table 1, the total amount of boron contained in the plant varies from 26.0 mg (stems of lupine), to 55.3 (flower-buds of lupine) mg of boron per 1 kg of absolutely dry matter. The stems usually contain the least amount of boron; as to the boron content in the leaves and flowers, there can be scarcely noticed any such regularity.

After 4 extractions, from 7.2 to 22.1 per cent of the boron originally contained in the plant remained in the pulp. A considerable fraction of boron must, therefore, have passed into solution.

It was difficult to notice any regularity in the relation between the solubility of the boron-bearing compounds and the part of the plant analysed. Attention was attracted by the high solubility of the boron contained in the flowers of buckwheat harvested at the end of flowering; at the time of maturation the same was found to be true of the leaves. All the parts of the plant, either in lupine or buckwheat, when harvested at the beginning of bloom fix rather similar quantities of boron, *viz.* from 13.5 to 17.3 per cent in the case of lupine, and from 15.9 to 22.1 per cent in the case of buckwheat.

We have compared the solubility of the boron-bearing compounds with that of other compounds contained in the plant, *viz.* those containing phosphorus, potassium and calcium. Like in the case of boron, four extractions by decantation were made here, and the content of the elements studied was determined both in the extract and in the pulp. Leaves of the narrow-leaved lupine were analysed. The results are represented in Table 2.

Table 2

Comparative Solubility of Potassium, Calcium, Phosphorus and Boron in the Leaves of Lupine (in per cent and in mg/kg of absolutely dry matter)

	K_2O		CaO		P_2O_5		B	
	mg/kg	per cent	mg/kg	per cent	mg/kg	per cent	mg/kg	per cent
Extract . .	3.32±0.18	97.5	0.56±0.27	72.3	0.41±0.05	36.5	28.0±0.1	86.5
Pulp . . .	0.08±0.02	2.5	0.21±0.0	27.7	0.72±0.02	63.5	4.4±0.4	13.5
Total . . .	3.40	100	0.77	100	1.13	100	32.4	100

As may be seen from Table 2, the elements studied can be arranged in the following sequence as to their solubility: phophorus, calcium, boron, potassium. Boron should thus be classed with the elements forming rather readily soluble compounds, although it can never be fully isolated even after repeated extraction.

Conclusions. 1. When plant materials are extracted with hot water by the method of periodic extraction, the greater part of boron contained in the plant passes into solution. However, under these conditions from $^1/_5$ to $^1/_{10}$ of the boron cannot be transferred to the solution.

2. The principal mineral elements contained in the plant form the following sequence as to their solubility in hot water: phosphorus, calcium, boron, potassium.

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PLANT PHYSIOLOGY

ON THE CORRELATION BETWEEN THE REACTION OF FLOWERING TO THE NITROGENOUS FOOD AND THE PHOTOPERIODIC REACTION OF THE PLANT

By M. CH. ČAJLACHJAN

(Communicated by A. N. Bach, Member of the Academy, 31. XII. 1944)

It has been shown in our previous notes ([2–6]) that the plant species studied by us can be classed into the three following main groups, according to their flowering reaction to nitrogenous food: 1) Nitrogen-negative plants which bloom earlier when nitrogen is completely eliminated or its dose is below the normal one, the situation being similar in the case of soils poor in nitrogen. 2) Nitrogen-positive plants which bloom earlier with normal or increased doses of nitrogen or upon soils rich in nitrogen. 3) Nitrogen-neutral plants which bloom at the same time, irrespective of the dosage of nitrogen or upon various soils.

Such a classification does not bear accidental character, but is undoubtedly connected with the physiological peculiarities of the development processes. We noted as early as in 1940 ([1]) that there exists a correlation between the photoperiodic reaction of the plants and the reaction of flowering to mineral nutrition: oats, a species of long day, produced ears and bloomed the earlier, the less was the dose of the elements of the mineral nutrient mixture; on the other hand, millet, a plant of short day, entered the stage of panicle formation and of flowering the earlier, the higher was the substratum in elements of mineral nutrition. By growing plants in water cultures Denffer ([7]) also found that species of long day, wheat, barley and iberis, were induced to earlier earing and blooming by eliminating nitrogen; on the other hand, the plants of short day, mohar, chrysanthemum, *Tinantia fugax* and *Kalanchoe Blossfeldiana*, were detained in flowering under the same conditions.

From a comparison of the data obtained by us ([6]) on the reaction of flowering of thirty species to nitrogenous food with their photoperiodic reaction, it is evident, on the one hand, that the nitrogen and photoperiodic groups mostly coincide, and, on the other hand, that there exist certain deviations from this regularity. Within group 1 of nitrogen-negative plants, wheat, barley, oats, mustard, spinach, alfalfa, clover, clarckia, geranium and iberis are plants of long day, whereas salvia is a plant of short day. Within group 2 of nitrogen-positive plants, millet, Indian corn, mohar, perilla, sunflower, tobacco, cotton, pepper, chrysanthemum, xanthium, *Tinantia fugax* and *Kalanchoe Blossfeldiana* are plants of short day, while lettuce, lupine and *Tagetes* are plants of long day. In group 3 of nitrogen-neutral plants buckwheat and bean are neutral with respect to daylength, while hemp and soy bean are plants of short day. Thus, from the total of thirty cases, in 24 cases there is a coincidence of the two types of reaction, the remaining 6 cases being aberrant.

In order to find out the cause responsible for these aberrations and to study in some detail the relations between the reaction of flowering to the nitrogenous

nutrition and the photoperiodic reaction, we have arranged, under the co-operation with L. P. Zhdanova, in 1944 experiments on 17 species at the greenhouse of the Institute of Plant Physiology. The following plants of long day were tested: 1) summer early wheat «Novinka», *Triticum vulgare* v. *erythrospermum*, 2) summer late wheat «Carosella», *Triticum vulgare* v. *lutescens*, 3) spinach «Vitofic», *Spinacia oleracea*, 4) opium poppy «Tar-Bagatai», *Papaver somniferum*, 5) raddish

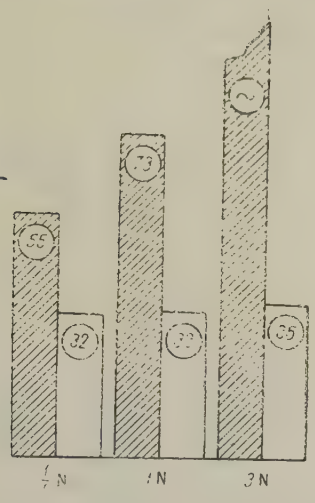

Fig. 1. Effect of nitrogenous nutrition upon flowering of long-day nitrogen-negative plant, early wheat «Novinka». L sets are shown by white bars, S+L variants, by shaded bars

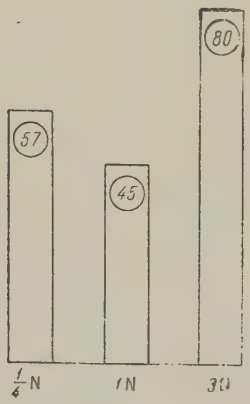

Fig. 2. Effect of nitrogenous nutrition upon flowering of a long-day nitrogen-positive plant of low threshold, raddish «Icicle», in the natural long day

«Icicle», *Raphanus sativus*, 6) blue lupine, *Lupinus angustifolius*, 7) *Rudbeckia hyrta*, 8) *Carthamus tinctorius*, 9) flax «Victor», *Linum usitatissimum*; and the following plants of short day: 10) *Cosmos* sp., 11) tomato «John Bear», *Solanum lycopersicum*, 12) Indian corn, *Zea mays*, 13) tobacco «Maryland 1460», *Nicotiana tabacum*, 14) *Hibiscus cannabinus*, 15) sunflower «Saratov 169», *Helianthus annuus*, 16) hemp «Novgorod-Seversk», *Cannabis sativa*, 17) hemp southern variety «Italian», *Cannabis sativa*.

The experiments were conducted in sand cultures, under application of the Prjanischnikow's nutrient solution in glass jars of the capacity of 4 kg sand. The experiments were arranged with varying amounts of ammonium nitrate, the scheme being as follows: 1) $^1/_4$ of the full dose of nitrogen ($^1/_4$ N), 2) one full dose of nitrogen (1 N), 3) three full doses of nitrogen (3 N); one full dose was introduced prior to sowing, the second and third doses being given in the form of additional nutrition on the 6th of July and on August, 11.

Sowing was done on June, 15. *Rudbeckia*, *Cosmos*, tomato and tobacco all were transplanted on the same day into the jars from the wooden boxes where the young plants were raised. The moisture of the sand was kept at 70 per cent of the full moisture content.

Under such varying conditions of nitrogenous food all the experimental plants were grown: 1) in natural long day (L), 2) in 10-hour short day (S), which was attained by driving the plants placed on waggons into the dark photoperiodic house from 6 o'clock in the afternoon until 8 o'clock of the next morning. Each experiment was run in double replication.

Since no reproduction took place under unfavourable photoperiodic conditions, all the long-day species, except for flax, were transferred from that day to the natu-

Effect of Nitrogenous Nutrition upon Reproductive Development of Plants of Long and Short Day

Plant species	Daylength	Number of days from germination to budding (earing) 1/4 N	1 N	3 N	flowering 1/4 N	1 N	3 N	Group with reference to flowering reaction to nitrogenous food
1. Long-day plants								
Early wheat «Novinka»	L	28	29	30	32	33	35	Nitrogen-negative
	S+L	49	68	79	55	73	none	
Late wheat «Carosella»	L	67	66	79	75	73	84	Do
Spinach «Vitofik»	L	—	—	—	14	14	15	»
	S+L	63	67	74	67	72	75	
Opium poppy «Tar-Bagatai»	L	55	64	—	58	72	—	»
Raddish «Icicle»	L	50	36	51	57	45	80	Nitrogen-positive, low threshold
Blue lupine	L	none	53	56	none	62	69	Do
Rudbeckia	L	46	34	36	76	63	67	»
Carthamus tinctorius	L	44	36	36	79	66	68	»
	S+L	74	70	74	—	—	—	
Flax «Victor»	L	34	34	35	42	39	40	Nitrogen-neutral
	S	49	47	47	54	53	54	
2. Short-day plants								
Cosmos	S	25	23	23	35	31	33	Nitrogen-positive, low threshold
	L+S	76	69	73	89	84	85	
Tomato «John Baer»	S	32	29	29	46	41	40	Nitrogen-positive
	L	44	29	32	none	41	53	Nitrogen-positive, low threshold
Indian corn (female flowers)	S	28	26	26	»	70	60	Nitrogen-positive
	L	42	27	29	»	81	80	
Indian corn (male flowers)	S	same			64	48	50	Nitrogen-positive, low threshold
	L				77	64	66	
Tobacco «Maryland 1460»	S	86	79	72	none	99	88	Nitrogen-positive
Kenaf	S	29	29	29	59	57	58	Do
	L+S	73	49	47	none	104	74	
Sunflower «Saratov 169»	S	28	23	23	55	47	41	»
	L	36	27	25	77	61	57	
Hemp «Novgorod-Seversk» (male plants)	S	14	14	14	24	23	23	Nitrogen-neutral
	L	22	22	21	29	32	33	
Hemp «Italian» (male plants)	L+S	53	53	53	66	66	66	Do

ral long day (S + L), and *Cosmos*, hemp (southern) and kenaf were transferred from long day to short day (L + S).

In the course of the experiments records were carefully taken of the developmental phases, of the dynamics of growth, as well as morphological analysis was done; at the close of the experiments (in October) the dry weight of the roots, of the aboveground vegetative and reproductive organs was determined.

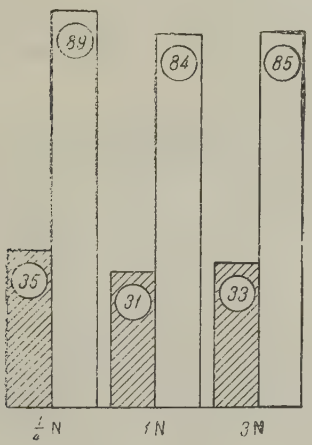

Fig. 3. Effect of nitrogenous nutrition upon flowering of a short-day nitrogen-positive plant of low threshold, *Cosmos*, in a short day (shaded bars) and in long plus short day (white bars)

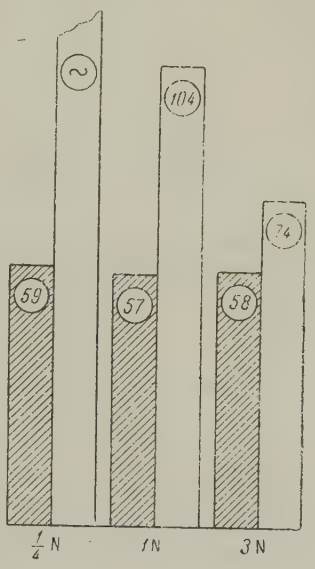

Fig. 4. Effect of nitrogenous nutrition upon flowering of a short-day nitrogen-positive plant, kenaf, in a short day (shaded bars) and in long plus short day (white bars)

The principal results of the experiments on the influence of the nitrogenous nutrition on the budding and flowering of the plants belonging to different photoperiodic groups are presented in the adjoining table and in Figs. 1—4.

It may be observed from the table that within the group of long-day plants early wheat, late wheat, spinach and poppy break into blossom the earlier, the smaller the dose of nitrogen, *i. e.* they are nitrogen-negative. Fig. 1 refers to this plant group; the number of days from germination to flowering is represented by vertical bars. Other long-day plants, raddish, lupine, *Rudbeckia* and *Carthamus*, are nitrogen-positive plants, but are characterized by a low threshold, as they attain flowering earliest when given one dose of nitrogen, flowering being detained both with $^1/_4$ and 3 doses of nitrogen (Fig. 2).

Within the group of short-day plants, *Cosmos*, tomato (with long day) and Indian corn (male flowers) similarly behave as nitrogen-positive plants of low threshold, for they bloom earliest with one dose of nitrogen, flowering being detained both by decreasing and increasing the dose (Fig. 3). Tobacco, kenaf, sunflower, tomato (with short day) and Indian corn (female flowers) flower the earlier, the greater the dose of nitrogen present in the substratum, *i. e.* display the typical behaviour of nitrogen-positive plants (Fig. 4).

Finally, flax and the two varieties of hemp flowered nearly at the same time with different nitrogen nutrition, thus forming the group of nitrogen-neutral plants.

All the nitrogen-negative plants are thus long-day plants. The nitrogen-positive plants of low threshold form an intermediate group as being at the same time plants

of long and short day. All the nitrogen-positive plants of high threshold are short-day plants. Finally, the nitrogen-neutral group includes both short-day and long-day species. All these data point to a considerable correlation between the nitrogen groups and the photoperiodic groups. The occurrence of an intermediate group of nitrogen-positive plants of low threshold, containing both short-day and long-day species, points to the existence of transitional forms between the typical nitrogen-negative and nitrogen-positive plants, a phenomenon which was earlier observed when classifying species into photoperiodic groups.

The aberrant cases found earlier in our studies ([3],[6]), *viz.*, the cases of lupine, lettuce and *Tagetes*, can be accounted for from the above point of view; they are nitrogen-positive plants of low threshold. Only the group of nitrogen-neutral plants, including photoperiodically neutral plants (buckwheat, bean), plants of short day (soy bean, hemp) and plants of long day (flax), is the most difficult to explain in virtue of the fact that the reaction of flowering to daylength and nitrogenous food coincides in a large number of species.

Two other circumstances connected with the reaction of flowering to nitrogen nutrition should be noted: it depends on the length of the growing period of the variety in question and on the earliness or lateness of the varieties within the same species. The greater the length of the growing period of a variety, as controlled by the light regime, the greater the differences in the rate of development as connected with the conditions of nitrogen nutrition. In early wheat they are greater with the short day plus long day, S + L (Fig. 1), in kenaf, with the long day plus short day, L + S (Fig. 4).

From the evolutionary point of view, the specificity of the flowering reaction of the different species to the nitrogen nutrition and the correlation of this reaction with the photoperiodic reaction may be understood when considered as an adaptation. Short-day plants, which are at the same time thermophilic ones, are immigrants from or inhabitants of southern tropical or subtropical regions where the growing period is not limited (or but slightly limited) by unfavourable conditions, and where adaptation to reproductive development and to preservation of progeny went along the line of maximum utilization of the conditions of nutrition, nitrogenous nutrition included. Long-day plants which are at the same time the most cold-resistant ones, are inhabitants of temperate and northern latitudes, where the growing period is strictly limited by the short summer; here the adaptation of reproductive development and the selection of species proceeded along the line of the fastest, and, consequently, of the slighter or slightest use of the conditions of nutrition in general, the nitrogenous nutrition in particular.

In such cases, where a lack of nitrogen would mean death, without leaving offspring, to nitrogen-positive short-day «warm-blooded» species, the nitrogen-negative long-day «cold-blooded» species have time enough to develop flowers and fruits, thus giving birth to a new generation.

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REFERENCES

[1] M. Ch. Čajlachjan and E. K. Lukovnikov, C. R. Acad. Sci. URSS, XXII, No. 2 (1941). [2] M. Čajlachjan, Bull. Acad. Sci. URSS d'Arménie, No. 9—10 (1942). [3] M. Ch. Čajlachjan, C. R. Acad. Sci. URSS, XLIII, No. 2 (1944). [4] M. Ch. Čajlachjan, ibid., XLIII, No. 9 (1944). [5] M. Ch. Čajlachjan, ibid., XLIV, No. 8 (1944). [6] M. Ch. Čajlachjan, ibid., XLVII, No. 2 (1945). [7] D. Denffer, Planta, 31, H. 3 (1940).

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PHYTOPATHOLOGY

ON THE IDENTITY BETWEEN YELLOW OF KOK-SAGHYZ AND YELLOW OF ASTER AND ITS POSSIBLE RELATION TO BIG BUD IN TOMATO

By K. S. SUKHOV and A. M. VOVK

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At Kaunchi, near Tashkent, Klechetov (4) observed in 1936 a mass disease in kok-saghyz which consisted in verescence and sterility of the flowers. The alteration of the inflorescences, Klechetov says, is very similar to the virus disease of the *Solanaceae*, of Morning glory and of other plants, known by the name of «big bud». At the same time Chilikina found in Kara-Tau a similar disease of tau-saghyz which was termed by her «filled heads» (4).

Klechetov pointed out the serious harm caused by this disease to the commercial plantations of kok-saghyz.

In 1942 Rischkov (7), independently of the two named writers, described from Bashkiria a similar disease in kok-saghyz and some other *Compositae*, as well as in several representatives of other families. Rischkov ascribed this disease to a new virus and named it the yellow of kok-saghyz. He pointed out its similarity to the yellow of aster stressing at the same time certain differences between the two diseases, for instance, profound alterations in the inflorescences, which, in his opinion, based on literary data, is not typical of the yellow of aster. Besides, such typical hosts of the yellow of aster as *Callistephus* sp., *Lactuca*, *Lycopersicum*, *etc.* were not found to be susceptible to the disease. However, the problem on the identity of this disease remained unsolved for lack of experimental evidence, and we undertook this study upon suggestion by V. L. Rischkov.

In the summer of 1944 the yellow was widely spread in the vicinity of Moscow. Dandelion proved to be the most appropriate plant for studying the spread of the disease. In 1944 it was affected in some localities to 40—50 per cent. A focus of extremely strong infection was found at the nursery of the Botanical Gardens of the Academy of Sciences of the USSR. According to data by Yudin, at the breeding plantations of kok-saghyz of the Mikhnevo Experiment Station of the Institute of Rubber Plants, 10—15 per cent of the plants were affected by yellow.

The nursery of the Botanical Gardens proved to be an important station where we could follow up the dynamics of the epiphytotia of yellow and its distribution among the various hosts.

The infectious nature of the disease was proved by us with the aid of grafting experiments and experiments involving transmission by insects. To obtain grafts we have grown together segments of diseased roots of dandelion with sound ones, after the method of Prokofiev(3). Diseased roots served as stocks, healthy ones, as scions. An experiment embracing 16 grafts was started on August, 16. After the 18th day 3 scions were affected; a week later 4 scions more developed the disease.

Later on the disease appeared on the rest of scions, so that the transmission of the disease attained 100 per cent. In the control group which consisted of 41 plants not a single one was diseased till the end of October. The symptoms of the disease in the experiment were sharply pronounced and typical, *viz.* there was chlorosis, reduction and malformation of the leaf blade, lighter coloration of the tissues along the veins. No flowers were produced by scions.

In another series of experiments we have succeeded in finding the carrier of the disease, this was found to be *Macrosteles quadripunctata* Kbm. (see table).

Transmission of Yellow by *M. quadripunctata*

Experimental plants	Runs	Number of experimental plants	Number of plants diseased	Percentage of plants diseased	Number of control plants	Percentage of diseased plants in the control	Mode of infecting the carrier
Kok-saghyz . . .	I	35	14	40	62	0	Feeding on diseased plants
Dandelion	I	49	34	69.3	48	0	
Kok-saghyz . . .	II	38	4	10.5	—	—	Spontaneously infected
Dandelion	II	39	4	10.2	—	—	

The symptoms of the disease transmitted by the insects did not differ from that in grafting experiments.

The testing of the two other species, *Euscelis Schenki* Kbm. and *Aphrodes bicinetus* Schen., gave negative results.

As a result of observations carried on throughout the summer at the nursery of the Botanical Gardens, a great number of hosts of the yellow of kok-saghyz could be discovered. Identification was done on the basis of the presence of virescence of the flowers often accompanied by other indices, chlorosis and lighter coloration of the leaf tissues along the veins. Below is a list of succeptible plants arranged in families (asterisks stand for cases where the disease was described in the species for the first time in the USSR:*, by Rischkov ([6]), **, by Klechetov, ***, by Beketov ([1]), ****, by Yachevsky ([8])).

Campanulaceae: Campanula glomerata L., *Campanula persicifolia* L.
Caryophyllaceae: Dianthus sp.
Compositae: Bellis perennis L., *Callistephus* sp., *Cichorium intybus* L.***, *Cirsium oleraceum* Scop. *, *Chrysanthemum* sp. *, *Gaillardia* sp., *Helenium autumnale* L., *Heliopsis* sp., *Hieracium* sp., *Lactuca sativa* L., *Lactuca scariola* L., *Matricaria inodora* L., *Senecio jacobaea* L., *Sonchus oleraceus* L. *, *Taraxacum kok-saghyz* Rod. **, *Taraxacum officinale* Wigg. *, *Zinnia elegans* Jacq. *.
Crassulaceae: Sedum spurium M. B.
Labiatae: Lamium album L.
Leguminosae: Lathyrus odoratus L., *Trifolium repens* L.
Papaveraceae: Eschscholtzia sp., *Papaver nudicaule* L., *Papaver somniferum* L.
Plantaginaceae: Plantago major L. *.
Polemoniaceae: Phlox Drummondii Hook. *.
Polygonaceae: Rumex confertus Willd.
Ranunculaceae: Delphinium sp.
Rutaceae: Dictamnus fraxinella Pers.
Scrophulariaceae: Nemesia sp., *Verbascum* sp.
Solanaceae: Solanum tuberosum L. ****.

The above list coincides nearly in all points with the list of the hosts of yellow of aster contained in the book by Smith ([13]). If one takes into consideration that in this country the disease is transmitted by a representative of the same genus *Macrosteles* to which belong the carrier of yellow of aster in America, and that

the principal symptoms of the disease coincide with the manifestation of the yellow of aster, we feel authorized to identify the yellow of kok-saghyz with the yellow of aster (*Callistephus* virus 1 Kunkel), or to assume its causative agent to be a strain of this virus.

Smith points out that the virus of aster yellow occurs in Germany and in Ungaria, *i. e.* in countries which are in immediate vicinity of, or near the USSR. Judging by the data collected by Penzig ([12]) in his survey of plant teratology containing descriptions of the virescence of the flowers nearly in all the plant species we and the American writers were dealing with, it seems probable that the yellow of aster is spread in the rest of Western Europe, including England, Holland, France, Italy and Austria.

It should be noted that the problem as to the identity of the yellow of kok-saghyz with the yellow of aster acquires a wider importance nowadays.

In our experiments on the transmission of the yellow with the aid of *M. quadripunctata* to tomato we have obtained diseased plants displaying characters very similar to those of big bud: small leaves, general or marginal chlorosis of the leaves, folding of the lobes of the leaves along the main vein, accompanied with a characteristic bend. Later on young fruits of one of these plants began to crack, which was not infrequently observed in the case of big buds, and was described by Kosteff as an important symptom of the disease. The flowers which had developed on the three plants after the appearance of the symptoms on the leaves, perished as buds.

Tomato plants showing symptoms of big buds in the vegetative parts and possessing cracking fruits, were found also in the field. It is quite natural that the spread of big bud of tomato near Moscow cannot be identical with that in the South because of greatly different ecological conditions. In the vicinity of Moscow big bud remains «immature», developing only the initial stage of the disease.

We have found, moreover, at the nursery of the Botanical Gardens potato plants bearing similar symptoms of big bud upon their stems and leaves, and bearing along with this virescent flowers showing antholysis of a type which entirely coincided with that of big bud. Grafting of shoot from these plants to tomato caused in the latter characteristic symptoms of big bud, inclusive of antholysis. We invariably found upon such potato crops *Macrosteles quadripunctata*.

It should be noted that symptoms of big bud may also be found in other hosts of the virus. Thus, for example, anatomical alterations in the pedicle of the diseased dandelion, bearing proliferated inflorescences, show a great similarity to the flower stalk of tomato infected with big bud, as shown by Mikhailova ([2]). In the case of dandelion there is also a hypertrophy of the phloem and a strong development of the xylem. The number of vessels in a bundle attains 120 in diseased plants against 20—25 vessels in healthy ones. Such pedicles have no cavity, the mark is filled with parenchymatous cells and bears a great number of accessory cylinders formed by the phloem, surrounded with the vessels of the xylem and by sclerenchyme. Finally, the type of antholysis nearly in all the various plant species susceptible to yellow corresponds well to that observed in the case of big bud.

All these data led us to the conclusion that the yellow of kok-saghyz should be identified not only with the big bud of aster, but, according to all probability, with the big bud of tomato, too.

A recent paper by Hill ([10]) went to corroborate our conclusions. This writer has recently demonstrated that the virus of big bud of tomato, which is spread in Australia, infects up to 65 plant species belonging to 24 families, many of which coincide with our list. It should be added that the transmitter of the big bud in Australia proved to be a representative of the genus *Thamnotettix*, two species of which (*Thamnotettix geminatus* and *Th. montanus*) are active transmitters of the virus of the yellow of aster in America, along with *Macrosteles divisus*. It should be noted also that Dana ([9]) has recently described big bud of tomato in the USA.

Eutettix ficitis and *Empoasca devastens* may probably be included into the list of carriers of the virus; they transmit in India the virus of «small leaves» in tomato ([14]), which is closely related to or identified with big bud.

It is very probable that in this country, too, the species of *Thamnotettix* play the rôle of additional transmitters of yellow and in certain geographic zones may even be its main carriers.

It may be said even now that the virus of the yellow of aster is transmitted by a rather wide range of species, which cannot be said of this disease in general. This fact may probably be associated with the extraordinary «omnivorous» habit of the virus which attacks such a great number of various plant species.

We take the opportunity to express our gratitude to Prof. A. A. Zakhvatkin for the identification of the insects, and to I. G. Beilin and A. G. Markov for the assistance in the present work.

Conclusions. 1. The yellow of kok-saghyz is an infectious virus disease and may be transmitted from a diseased plant to a healthy one by grafting.

2. The transmitter of the virus is *Macrosteles quadripunctata* Kbm.

3. According to our own and literary data, the virus attacks in the USSR up to 45 plant species belonging to 15 families.

4. A comparison of the transmitters and hosts makes it possible to identify the virus of yellow of kok-saghyz with the virus of the yellow of aster (*Callistephus* virus 1 Kunkel) which is spread in America, or to assume it to be a related strain.

5. A similar comparison reveals a close affinity, if not an identity, between the virus described and the causative agent of big bud of tomato (*Lycopersicum* virus 5 Bald., Samuel and Fardley, Rischkov).

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REFERENCES

[1] А. Бекетов, Тр. С.-Петерб. общ. естествоисп., 8 (1877). [2] П. В. Михайлова, Вирусные болезни растений в Крыму и на Украине, Симферополь, 1934. [3] A. A. Prokofiev, C. R. Acad. Sci. URSS, XLIV, No. 4 (1943). [4] Промышленные каучуконосы СССР, М. стр. 406, 1938. [5] В. Л. Рыжков, Вирусные болезни растений в Крыму и на Украине, Симферополь, стр. 59, 1934. [6] В. Л. Рыжков, Сб. памяти Тимирязева, стр. 696, М., 1936. [7] V. L. Rischkov, C. R. Acad. Sci. URSS, XLI, No. 2 (1943). [8] А. А. Ячевский, Ежегодн. свед. о болезн. раст., 1910. [9] B. F. Dana, Phytopath., **30** (1940). [10] A. V. Hill, J. Coun. Sci. Industr. Res. Aust., **16**, No. 2 (1943). [11] D. Kostoff, Genetica, **15**, No. 1—2 (1933). [12] O. Penzig, Pflanzen Teratologie, 1921. [13] K. Smith, Textbook of plant viruses, 1937. [14] K. M. Thomas and C. S. Krishnaswami, Proc. Ind. Acad. Sci., **10**, No. 2 (1939).

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PHYTOPATHOLOGY

INFLUENCE OF APHIDES UPON CARBOHYDRATE EXCHANGE IN COTTON

By Z. L. NEVSKAYA

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The first attempt to inquire into the influence produced by leaf injuries upon the development of plants belonged to Lubimenko (1); this author succeeded in demonstrating that the harmful effect of these injuries is not confined merely to the reduction of the productivity of photosynthesis, but is accompanied by secondary processes, whose intensity should also be taken into account.

The experiments carried out in 1940 involved several cotton varieties* infested with the so-called melon aphis (*Aphis frangulae* Koch); during that year cotton crops were badly affected with this pest, both with regard to the number of plots infested, density of population and period of activity of the pest. Part of the work was also carried out with cotton infested with the big cotton aphis (*Acyrthosyphon gossypii* Mordv.).

The degree of infestation of the leaves with aphides was estimated visually; in case the lower surface of a leaf blade was covered with aphis colonies to no less than 50 per cent of its area, we considered it sufficiently infested for our purposes.

The alterations that took place in the affected organs of the plants were studied against the background of and compared with the alterations that occurred in the corresponding organs of sound plants of the same age, variety, and grown under identical conditions.

Both for settling and sucking the melon aphides mostly prefer the lower side of the leaf blade; there are periods, however, especially during autumn, when they settle also on the upper side of the leaf blade, on petioles, perianth and buds. As to *Acyrthosyphon gossypii*, under natural conditions these aphides are but rarely found upon leaves, to which they usually prefer petioles and young shoots. We had therefore recourse to cambric insulators which were put upon separate leaf blades after their artificial infestation with aphides.

Unlike other insects, the aphides cannot feed otherwise than on solved substances; the digestion of solid food begins therefore outside their organism, in the tissues of the plant they are feeding upon. The digestion is effected by means of enzymes excreted by the salivary glands, through the sucking apparatus (proboscis). This peculiar mode of sucking should, therefore, affect in the first line the enzymic system of the vegetable organism, telling upon the balance of its nutrient elements. The aim of the present study was to find out the disturbances caused by aphides in the carbohydrate metabolism of the cotton plant; the period of observation comprises the time interval following the first contact between the two enzymic systems, that of the parasite and that of the host; the subject of observation was the activity of amylase and invertase in the affected leaves.

* Uplands 8517 and 2034, and hybrids between the Indian variety, on the one hand, and the Peruvian and Egyptian, on the other.

Table 1

Activity of Amylase in mg of Maltose, and of Invertase in mg of Glycose, in Leaves of Cotton Plants Affected or Unaffected by Melon Aphis (per 1 g raw weight of leaves)

Date	Amylase									Invertase								
	Variety 8517							Variety 2034		Variety 8517								
	9.VI	13.VI	15.VI	15.VII	29.VII	30.VII	4.VIII	Mean %	17.VII	21.VII	22.VII	Mean %	9.IX	10.IX	18.IX	29.IX	11.X	12.X
Control	17.34	53.20	35.70	50.72	29.10	18.78	68.68	—	48 74	36.54	54.70	—	24	0	0	35.75	9.09	6.79
Affected	49.66	113.98	58.18	79.40	58.28	42.18	96, 92	—	66, 04	60.70	61.64	—	2950	238	402	50.0	30.45	16.94
Per cent to control	285	214	163	156	200	224	141	197	135	166	113	138	1229	—	—	140	335	250

The ground mass was infused during an hour at room temperature; after this the infusum was filtered off with the aid of a pump, divided into two equal portions, one of which was inactivized by heating for 15 minutes over a water bath. The two portions were added equal amounts of starch or saccharose, in order to determine the activity of amylase or invertase, respectively. The mixtures were allowed to stay 24 hours at a temperature of 27—30 °C, after which they were boiled during 5 minutes, filtered, and the content of reducing sugars was determined in the filtrate thus obtained.

The quantity of carbohydrates was determined in raw, air-dry material (previously inactivized by vapour), in sap expressed after Kiezel's simplified method, followed by a determination of reducing sugars, according to Issekutz. Comparative data on the activity of the hydrolysing power of amylase and invertase are summarized in Table 1.

The absolute values of amylase activity are invariably found in affected leaves to be higher than in intact ones; on the average, it is twice as high in the former as in the latter. A similar picture was observed in the case of cotton infested by *Acyrthosyphon gossypii* aphides where the activity of amylase in the affected plants was found to be 2.5 times that of the sound ones.

The values referred to above are subject to considerable fluctuations, which may depend upon a variety of conditions, and, in the first line, upon the density of the aphis population, the prevalence of individuals of a definite development stage, *etc.*

Thus, both aphis species, irrespective of their different mode of feeding described above, produce in the process of sucking in the tissues of leaves of different cotton varieties such conditions as are favourable for an increased hydrolysis of starch. In affected leaves the activity of both invertase and amylase is found to be higher than in intact ones. Even in such cases where no invertase could be discovered in healthy leaves, the affected leaves always showed an activity of invertase which could well be recorded.

The increase observed in the activity of invertase in affected leaves is evidence of that the aphides assimilate the carbohydrates largely in the form of monoses, and that not only starch, but soluble disugars, too, are previously split into monosugars under the influence of the excreta of their salivary glands.

We are thus brought to assume that in the course of their adaptation process the aphides have developed a certain capacity which plays a beneficial rôle in their relations with the host-plants.

The next step in our work was to find out the influence of the quantitative difference in the activity of carbohydrases upon the quality and quantity of the carbohydrates migrating and accumulating in plants affected by aphides.

The picture observed is similar in all cases (Table 2): the quantity of monosugars shows an average increase by 16 per cent, while that of saccharose falls to 54 per cent to its content in healthy leaves (an average decrease by 38 per cent).

An analysis of the sap expressed from petioles of affected and unaffected leaves gave the following results.

Table 2

Contents of Soluble Sugars in the Sap from Leaves and Petioles of Cotton (in mg per 100 cm³ of sap)

Date	Cotton variety	Monosugars			Disugars		
		Affected	Unaffected	Affected in % to sound	Affected	Unaffected	Affected in % to sound
		Leaf sap					
29.VIII	8517	730	550	133	181	352	51
1.IX	8517	780	720	108	314	684	46
2.IX	8517	766	679	113	140	199	70
7.IX	8517	648	596	110	443	544	80
Mean	—	—	—	116	—	—	62
		Petiole sap					
29.VIII	8517	570	840	68	578	808	71
1.IX	8517	860	1340	64	570	1359	42
2.IX	8517	272	1113	24	636	1128	56
3.IX	2034	333	677	50	417	757	55
7.IX	8517	596	760	78	901	1327	68
12.IX	119 *	800	1890	42	—	—	—
13.IX	— **	437	1360	32	—	—	—
18.IX	8517	288	1044	27	683	1093	63
Mean	—	519	1128	48	631	1079	59

* Hybrid between the Indian and the Peruvian varieties.
** Hybrid between the Indian and the Egyptian variety.

The data obtained point to a considerable decrease in the concentrations of the carbohydrates transported by the sap of the petioles of affected leaves, as compared to that of the healthy ones (by 48 per cent of monosugars and 59 per cent of saccharose); it should be added that the drop in the concentration of disugars is less pronounced than in monosugars.

The sap of the petioles is a product of the assimilation activity of the leaves, and must therefore to a certain extent illustrate the changes suffered by this process. Table 3 shows the contents of soluble sugars in the sap of petioles, determined during one day at 2-hour intervals.

The data obtained illustrate a regular decrease in the content of carbohydrates in the affected petioles. Their quantity is always much lower than in unaffected petioles, irrespective of the hour of the day, the average decrease being 75 per cent in the case of monosugars, and 42 per cent for disugars.

Table 3

Contents of Soluble Sugars in the Sap of Petioles, Determined During one Day in mg per 100 cm³ of Sap

Time in hours	Monosugars			Disugars			Ratio between sugars and disugars	
	Affected	Unaffected	Affected in % to sound	Affected	Unaffected	Affected in % to sound	Affected	Unaffected
6	272	1113	24	636	1128	56	0 43	0.94
8	447	1418	32	790	1545	51	0.57	0.89
10	336	1192	28	615	944	65	0 55	1.20
12	280	1150	24	722	1399	52	0.39	0.80
14	260	1124	23	865	1432	60	0.30	0.74
16	295	1335	22	776	1148	68	0.38	1.16
Mean	315	1222	25.7	734	1266	58.0	0.44	0.96

A comparison of sugar contents in the sap expressed from sympodial shoots of affected and unaffected leaves revealed an analogous picture, *viz.* the contents of sugars in affected sympodia is on the average by 65 per cent lower than in the sap of sound ones.

From the above comparative analysis of stem and root it is obvious that the contents of reserve, readily mobilized carbohydrates—starch and saccharose — is considerably reduced in the stem and root of cotton plants infested during 3 months with melon aphides (Table 4).

The investigation here reported permits of the following conclusions:

1. In leaves of cotton plants infested with melon and cotton aphides which in the process of sucking introduce into the tissue of the leaf some enzymes of their

Table 4

Contents of Monosugars, Disugars and Starch in Stems and Roots of Cotton Plant Variety 8517 Infested with Melon Aphis

Plant organ	Date	In per cent to dry weight						In per cent to air-dry weight		
		Monosugars			Disugars			Starch		
		Affected	Unaffected	Affected in % to sound	Affected	Unaffected	Affected in % to sound	Affected	Unaffected	Affected in % to sound
Stem, top	21.IX	1.47	2.02	73	0.52	1.54	34	—	—	—
	23.IX	1.37	2.12	65	0.31	1.50	21	3.81	7.92	48
Mean		1.42	2.07	69	0.42	1.52	27	—	—	—
Stem, bottom . .	21.IX	1.05	2.03	52	1.48	3.36	44	—	—	—
	23.IX	1.49	2.05	73	0.92	1.93	48	6.97	11.49	61
Mean		1.27	2.04	62	1.20	2.64	45	—	—	—
Root	21 IX	1.05	1.07	98	3.36	3.83	88	—	—	—
	23.IX	1.32	1.31	100.8	1.95	2.75	71	8.44	18.27	46
Mean		1.19	1.19	100.0	2 65	3.29	80.6	—	—	—

salivary glands, the activity of amylase and invertase is increased by 100 per cent on the average.

2. Owing to intense hydrolysis, di- and polysaccharids pass into the only form in which they can be directly assimilated by aphides, *viz.* into monosugars.

3. The monosugars formed are sucked by the aphides so intensely that the concentration of soluble carbohydrates in the sap of petioles, which are the first to convey the downward flow of the sap, is reduced by 40 per cent on the average.

4. The amount of reserve carbohydrates in the axial organs of the cotton plant is reduced by about 50 per cent.

5. The carbohydrate starvation of the vegetative organs leads to a poor supply of the fructification organs with nutrient material, which results in their shedding at the critical periods of growth of the cotton plant, and also affects the quality of raw cotton in the bolls that escaped shedding.

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REFERENCES

[1] В. Н. Любименко, Тр. совещания по защите растений, III, в 3. (1933).

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EXPERIMENTAL MORPHOLOGY

INDUCTION OF TYMPANIC MEMBRANE IN *BOMBINA BOMBINA* AND *PELOBATES FUSCUS*

By V. V. POPOV

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The morphological aspect of evolutionary process lies largely within the domain of comparative morphology. The latter, however, leaves out of account the latent possibilities of morphogenesis which, as it will be shown below, must play an important rôle in the working out of a natural system. Mangold's investigation (1) on the development of the balancers may serve as an example to illustrate this statement. In this case «reacting system» has been affected by evolutionary change, while the source of the formative influences has been left without noticeable changes.

I was much interested in the problem of evolution of «inductor» and «reactor» in their possible interaction, and have more than once suggested the desirability of studying this problem on material connected with the reduction of the eye (2). For lack of suitable experimental material on eye development, I paid attention to the development of the tympanic membrane which in number of representatives of batrachians is reduced to a considerable degree, or even completely lost.

In his interesting investigation (4) Helff was the first to demonstrate the inductive influence in the development of the tympanic membrane. He has found that the epidermal part of the tympanic membrane (or, shortly speaking, the tympanic membrane) develops under the influence of inductive action of the tympanic cartilage (*annulus tympanicus*), ring-like in shape, surrounding and at the same time underlying the edges of the tympanic membrane.

The tympanic cartilage and, consequently, the tympanic membrane are well expressed in batrachians and especially in *Rana esculenta* and *Rana ridibunda*. In *Bufo bufo* the diameter of the tympanic membrane is small and highly variable, while in *Pelobates fuscus*, according to literary data, «the tympanic membrane is hidden beneath the skin». Apparently, the tympanic cartilage and the basic part of the tympanic membrane drawn on the cartilage are understood in this case. In *Pelobates fuscus* the dermal part of the organ in question, which we call together with some other authors, briefly, «the tympanic membrane», is completely lost. At last, in *Bombina bombina* (a representative of *Discoglossidae* family) the tympanic membrane and the tympanic cartilage are completely missing.

In connexion with the problem formulated above the experiments were planned with special reference to the evolution of the tympanic membrane in *Bombina* and *Pelobates* as well as to the latent possibilities in its development. The problem as to the formative peculiarities of the «reacting system» was the first to be studied in the present investigation.

The skin from *Bombina* and *Pelobates* tadpoles of different ages was transplanted to young *Rana esculenta*, *R. ridibunda* and partly *R. temporaria* in place of the tympanic membrane removed. The age of the donors varied from the time of the appearance of leg rudiments to the stage just preceding that of the liberation of the fore limbs. The hosts were, in most cases, in their first year, their tail being already reduced by half, or completely. The transplantations were carried out in all the possible combinations as to the age of the donors and hosts. All the experimental animals were, consequently, divided into definite groups. There is no necessity to consider them separately, since the induction of the tympanic membrane took place in all of them. Some transplantations made on the animals of similar age (the host and the donor were in their first year) gave obscure and, apparently, negative results.

In addition, a number of additional experiments were carried out in which *Bombina bombina* animals were used as hosts. In these experiments the tympanic membrane with the connective-tissue layer taken from the young *Rana esculenta* and *R. ridibunda* was transplanted beneath the back skin of young *Bombina* tadpoles. The development of the tympanic membrane also took place by the end of metamorphosis. It should be pointed out, however, that part of the experiments led to negative results. Moreover, the membranes developed in part of the successful experiments were found to be differentiated not so perfectly as those observed in the main series of experiments. These results are, on my opinion, due to resorption and deformation of the transplants.

The experiments on transplantation of the head skin of *Bombina bombina* and *Pelobates fuscus* tadpoles in place of the tympanic membrane of 1—2-year-old and adult *Rana esculenta* and *R. ridibunda* frogs stand somewhat apart as to the technique applied. It was found that even the definitive tympanic cartilage did not lose its formative ability, and induced a perfect tympanic membrane. The same phenomenon has been found by Bimbad to be true of the other species. It is interesting to note that, as shown by special experiments, even in adult animals the regeneration of the membrane was also brought about under the influence of induction. These facts seem to indicate that the definitive cartilage possesses the inductive ability for which natural selection must be taken responsible. Of course, the tympanic membrane, being located rather superficially, is subject to injuries under natural conditions. Regeneration with subsequent induction is the most economical way to restore the organ lost.

A total of 81 successful transplantations were carried out in which the experimental animals survived during a necessary time. The skin of the *Bombina* tadpoles was transplanted to young *Rana temporaria*, young *R. esculenta* and *R. ridibunda*, 1 or 2-year-old frogs of the latter species, and to adult animals of the same species, the number of transplantations being 4, 32, 3, and 4, respectively. The skin from *Pelobates* tadpoles was transplanted to the young *Rana esculenta* and *R. ridibunda* and to adult animals of the same species, the number of transplantations being 9 and 5, respectively. In 10 cases the tympanic cartilage of the young *Rana esculenta* and *R. ridibunda* was transplanted beneath the skin of *Bombina* tadpoles. Finally, 17 experiments were carried out on regeneration of the tympanic membrane in *Rana esculenta* and *R. ridibunda* at different ages.

As shown by binocular observations on the operated animals, the skin transplanted, first, lost a large part of the pigment, then became vascular, and, finally, regained its pigment, this latter being, however, peculiar to the tympanic membrane. It is interesting to note that the skin of *Bombina* is changed somewhat earlier than that of *Pelobates*.

In summer and early in autumn fully developed tympanic membrane is developed under the influence of the tympanic cartilage of young tadpoles within 30—40 days. Under the influence of tympanic cartilage of 1—2-year-old frogs the organ in question is developed within the same period. If the inductor belongs to adults, the time of development increases to 45—60 days. The formation of the tympanic membrane in the experiments on regeneration is complete somewhat earlier, especially in young animals.

The suggestion as to a gradual replacement of the skin transplanted by the host's epidermis and, consequently, to the erroneousness of the results of our experiments should be discarded because of the following facts. The pigmentation of the transplant is different from that of the host, which is in most cases fairly well seen till to the end of experiments. The same was found to be true in the case where the skin of the donor had previously been marked by carmine ([2]). The arrangement of the carmine granules remained unchanged up to the end of the experiments. The transplants were different from the hosts in respect to the size of their nuclei. Measurements of the latter and their biometrical treatment showed them to keep the differences in question up to the end of experiments. Finally, the very fact of induction of the tympanic membrane, be it in the skin of *Bombina bombina* alone, was established quite definitely not only by means of skin transplantation, but also in the experiments on the transplantation of the tympanic cartilage itself, when the resorption and replacement of the reactor is out of the question.

The preliminary data obtained from the histological examination of the material fixed at different periods went fully to confirm the pictures observed on live material. The skin of *Bombina bombina* and *Pelobates fuscus* is transformed into a true tympanic membrane. Therefore, in spite of the fact that the ancestors of the recent *Pelobates fuscus* and *Bombina bombina* have lost the tympanic membrane and tympanic cartilage since a very long time, their skin is still keeping the ability to transform into a tympanic membrane. These facts seam also to indicate that reducing changes of the tympanum in the species under consideration are conditioned historically.

Another aspect of these experiments is connected with the problem of species specific to the action of different formative inductors. It is commonly adopted that the former is determined only by the species peculiarities of «the reacting system». The data described above show conclusively that this principle is wrong when applied in its universal interpretation. Indeed, we were successful in obtaining the tympanic membrane in *Bombina bombina*, in the species diagnosis of which the part of the tympanum is completely absent. This case seems to be the first where the organ developed was «the inductor» rather than «the reactor», for this is the interpretation to be given to this case from the point of view of modern ideas. True enough, the definition of «species specificity» and «non-species specificity» in development mechanics are worked out rather superficially, and stand in need for further elaboration.

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REFERENCES

[3] O. Mangold, Nature, **19** (1931). [2] В. В. Попов, Биол. журн., **3** (1934). [3] В. В. Попов, Тр. кафедры Гист. и эмбр. Горьк. гос. ун-та, 1 (1940). [4] O. M. Helff, Physiol. Zool., 1 (1928).